Suppression of Multiple Narrowband Interferers in a Spread-spectrum Communication System*†

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Abstract

We consider the problem of estimating and suppressing many unknown independent and time-varying interferers in a spread-spectrum communication system. The interferers are assumed to be present in a wide frequency range. In order to detect, estimate and track the interference, we use a bank of hidden Markov model filters operating in the frequency domain. The hidden Markov model filters' outputs are then used to suppress the existing interference. The computational complexity of our scheme is only linear in the number of interferers. The simulation studies show that our proposed novel schemes adapt quickly in tracking the time-varying nature of the interference.

1 Introduction

In this paper, we consider the problem of detecting, tracking and suppressing interference in a spread-spectrum communication system. The interference is assumed to consist of many unknown, independent and time-varying narrowband interferers.

There are several advantages with spreading the spectrum of the signal that you wish to transmit; see *e.g.* [2, 3]. One of the major advantages is the inherent ability to reject interfering signals whose bandwidths are small compared to that of the spread spectrum. However, the interference may be powerful enough so that communication becomes effectively impossible. In order to improve the performance of a spread-spectrum communication link in an interference corrupted environment without increasing transmit bandwidth (or reducing throughput), some additional means of interference removal must be used. The problem of interference suppression

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has been studied extensively over the last decades [2, 3, 4]. Previous work in this area can be classified into frequency domain and time domain approaches.

Time domain techniques for narrowband interference can be split into linear and non-linear methods. [2] provides a discussion on parametric linear techniques for narrowband interference suppression. In these linear techniques, also known as estimator/subtracter methods [5], a transversal filter is used to obtain estimates of the received signal based on previous samples and model assumptions. The filter is implemented using single-sided taps (linear prediction filter) or double-sided taps (linear interpolation filter). Interpolation linear filters were found to give greater interference suppression. Adaptive schemes based on the least mean squares (LMS) algorithm and lattice filters have been developed [6, 7, 8]. Different models for narrowband interference exist in the literature; *e.g.* in [8, 9], the interference is modeled as a Gaussian AR process, in [7] as a sum of sinusoids and in [6] as a pulsed RF tone. In 1991, Vijayan and Poor [10] suggested a non-linear technique for prediction of the narrowband interference signal that took into account the non-Gaussian distribution of the observation noise. This led to various nonlinear techniques to combat narrowband interference suppression in spread spectrum systems [5, 10, 11]. The interference in [5, 10, 11] is modeled as a Gaussian AR process. For known interference statistics the interference is estimated using an approximate conditional mean (ACM) filter [12]. The ACM filter is a modification of the Kalman filter that deals with non-Gaussian distributions in the observation or the state process. For the specified assumptions on the observation process in [5, 10, 11], the ACM filter for interference estimation turns out to be a Kalman-type recursive filter which includes some nonlinearities. In [13], a new (suboptimal) nonlinear filter and parameter estimator for narrowband interference suppression in spread spectrum systems is presented. A cross-coupled hidden Markov model (HMM) and a Kalman filter were used to compute the desired state estimates. In [14], no assumptions on the statistics of the involved noise processes are made. An iterative scheme based on the coordinate descent method is proposed where the spread-spectrum signal is estimated using the Viterbi algorithm while the interference is estimated in an l_1 sense using an interior point based scheme.

Frequency domain techniques are usually non-parametric and require no prior knowledge of the characteristics of the interference. These algorithms are based on transform domain filtering;

see *e.g.* [7, 15, 16]. The key idea is that the received signal is estimated and used to design filters that attenuate the signal in the frequency range where the interference is dominant. [7] employs the fast Fourier transform (FFT) algorithm for performing spectral analysis on the received signal, and on the basis of the spectral estimate, a transversal filter is designed for suppressing the interference. In [15, 16], real-time Fourier transformation using a surface acoustic wave (SAW) with a chirp impulse response built into the taps is used. In [17], the problem of suppressing many independent narrowband interferers that are present in different frequency bins is studied. Adaptive filtering in the frequency domain via the LMS algorithm is performed in order to track and estimate the interference.

In this paper, we consider a similar problem to the one posed in [17]. In this paper, we assume that there are many *possible* frequencies at which interferers might be present. The *maximum* number of possible frequencies at which interferers may exist can be chosen very large in order to detect all interferers. Each interferer has an on-off keying modulation and is dedicated a fixed phase and is purely single frequency sinusoidal. The presence and statistics of the interferers are highly time-varying, *i.e.*, an interferer is allowed to appear, disappear and possibly be replaced by another interferer with different statistics. This model is physically more realistic and flexible than a model assuming fixed interference. By modeling the amplitude of each narrowband interferer with an independent Markov chain, the possible presence of the interferers is described efficiently.

We use a bank of independent HMM estimators operating in the frequency domain to detect and estimate the interference. Each of the HMM estimators yields the conditional mean estimate of a narrowband interferer. The estimated narrowband signals are then removed prior to transforming back to time domain. By detecting, estimating and suppressing the existing multiple narrowband interferers in the frequency domain instead of in the time domain, we make use of the fact that the interferers are independent in the frequency domain. We can therefore apply an adaptive estimator to each of the interferers independently and thereby, the computational complexity of detecting the narrowband interferers is linear in the number of interferers instead of exponentially growing which would have been the case if the estimation of the interference would have been performed in the time domain. This is the case since we in the time-domain approach would have to investigate all possible combinations of active and inactive interferers. Keeping the computational complexity as low as possible is crucial since the number of interferers can be very high.

The outline of this paper is as follows. In Section 2, we describe the signal model and state the objectives. Next, in Section 3, we propose and analyze off-line and on-line schemes for suppression of multiple narrowband interferers. Section 4 gives some experimental results and finally, Section 5 provides concluding remarks.

2 Signal Model and Objectives

In this section, we describe the model for the received noise corrupted signal and we then state our estimation objectives.

Continuous-time Signal Model

We assume that the continuous-time, complex envelope of the received signal, $y(t)$, consists of the sum of the spread spectrum signal $s(t)$, the interference $x(t)$, and the observation noise $w(t)$, *i.e.*,

$$y(t) = s(t) + x(t) + w(t). \tag{1}$$

The interference $x(t)$, is, similar to [6], assumed to consist of at the most M superimposed independent narrowband interferers, $x^m(t),\ m = 0, 1, \dots M-1$, where M is a known integer, *i.e.*

$$x(t) = \sum_{m=0}^{M-1} x^m(t), \tag{2}$$

where

$$x^m(t) = \xi^m(t) e^{j2\pi mFt},\ m = 0, 1, \dots M-1, \tag{3}$$

where $\xi^m(t)$ is an unknown stochastic random process indicating the signal strength and presence of the mth interferer at the known frequency mF.

The spread spectrum signal $s(t)$ is assumed to be transmitted over the entire frequency range $[0, f_{max}]$, where $f_{max} = (M-1)F$, while the mth narrowband interferer, $x^m(t)$, operates at frequency mF, for $m = 0, 1, \dots M-1$. Thus, the interferers are assumed to be equispaced in frequency.

Discrete-time Signal Model

The discrete-time model is the result of sampling the continuous-time model in (1) at sampling rate $1/T_s$. The discrete-time observations are given as follows

$$y_k = s_k + x_k + w_k, \tag{4}$$

where s_k is a sampled spread spectrum signal, w_k is a zero mean white circular Gaussian process with variance σ_w^2 and the interference x is given by

$$x_k = \sum_{m=0}^{M-1} \xi_k^m e^{j2\pi mFkT_s}. \tag{5}$$

The sampling period, T_s, is assumed to be equal to the chip period, T_c, of the spread spectrum signal. The condition $T_s = T_c$ ensures the sampled spread spectrum signal s_k to be an independent identically distributed (iid) process. The process $\xi_k^m,\ m = 0, 1, \ldots M-1$ is assumed to be constant during the time batches $bN - N, \ldots bN - 1$ for $b = 1, \ldots T$. It is denoted as $A_b^m,\ b = 1, \ldots T$. The amplitude of the mth interferer, $A_b^m,\ m = 0, 1, \ldots M - 1$, is modeled as a discrete-time, homogeneous, first-order, 2-state Markov chain with states $q^m = (q_1^m, q_2^m)$, where $q_1^m = 0$ and q_2^m is an unknown non-zero complex number, *i.e.* A^m serves as an indicator function for the presence of the mth interferer. Thus, when the mth Markov chain A_b^m is at state q_1^m, it implies that there is no interferer present at frequency mF. On the other hand, when A_b^m is equal to q_2^m, then the interfering amplitude is q_2^m. The transition probability matrix of A^m is denoted by $\Pi^m = (\pi_{np}^m)$, where π_{np}^m denotes the probability that A_{b+1}^m is in state q_p^m given that A_b^m was in state q_n^m. That is,

$$\pi_{np}^m = \mathrm{P}\left(A_{b+1}^m = q_p^m | A_b^m = q_n^m\right),\ n, p \in \{1, 2\}. \tag{6}$$

Of course, $\pi_{np}^m \geq 0$ and $\sum_{p=1}^{2} \pi_{np}^m = 1,\ \forall n, p \in \{1, 2\}$ and $m \in \{0, 1, \ldots M - 1\}$. Let $\bar{\pi}^m,\ m = 0, \ldots M-1$ denote the initial state probability vector that is the vector containing the probabilities of the states of A_1^m.

$$\bar{\pi}^m = (\bar{\pi}_n^m),\ \forall m \in \{0, 1, \ldots M - 1\},\ \text{where } \bar{\pi}_n^m = \mathrm{P}\left(A_1^m = q_n^m\right)\ \forall m \in \{0, 1, \ldots M - 1\}. \tag{7}$$

Objectives

Given the discrete-time observations y, the objective is to propose both off-line and on-line algorithms that

• are of low complexity. The computational complexity should increase linearly with the number of possible interferers.

• detect accurately which frequencies mF for $m = 0, 1, \ldots M-1$ are corrupted with interference. Using the model description in Section 2, this is equivalent to estimating the state of the Markov chain A^m for $m = 0, 1, \ldots M-1$.

• estimate accurately with what probability the mth interferer x^m, $m = 0, 1, \ldots M-1$, is present.

• estimate accurately the interferers x^m, $\forall m \in \{0, 1, \ldots M-1\}$. Using the model description in Section 2, this is equivalent to determining q_2^m, $\forall m \in \{0, 1, \ldots M-1\}$.

• estimate accurately the transition probabilities from absence to presence of the interferers, $i.e$ determine Π^m, $\forall m \in \{0, 1, \ldots M-1\}$ in our model description above.

• suppress the interference, x.

3 Algorithms for Interference Estimation and Suppression

In this section, we propose off-line and on-line schemes for detection, tracking, estimation and suppression of the multiple narrowband interferers in a spread-spectrum communication system.

The key idea of the suggested algorithms is to perform adaptive filtering in the frequency domain using a bank of hidden Markov model (HMM) estimators (filters for the on-line scheme and smoothers for the off-line scheme) to detect and estimate the interference. The outputs of the HMM filters/smoothers are then used to determine the parameters of our proposed suppression scheme which is also run in the frequency domain. We choose to perform the adaptive estimation in the frequency domain and not in the time domain in order to make use of the fact that the interferers are independent in the frequency domain. Therefore, the computational complexity of detecting the narrowband interferers is linear in the number of interferers instead of exponentially growing which would have been the case if the estimation of the interference would have been performed in the time domain. Keeping the computational complexity as a function of the number of interferers as low as possible is crucial since the number of interferers creating the interference can be very high. In Fig. 1, our proposed scheme is depicted. The output data are grouped into batches of T consecutive blocks. Each block contains N data points. The discrete Fourier transform (DFT) of the bth batch is given by

$$Y(kF_s, b) = T_s \sum_{n=bN-N}^{bN-1} y_n e^{-j\frac{2\pi n}{N}k}, \; k = 0, \dots N-1, \; b = 1, \dots T, \tag{8}$$

where $F_s = 1/(NT_s)$. The received signal is, as stated in Section 2, a sum of the transmitted spread spectrum signal, s, the observation noise, w, and the interference, x. The DFTs of each of these components are as follows.

$$X(kF_s, b) \;=\; T_s \sum_{n=bN-N}^{bN-1} x_n e^{-j\frac{2\pi n}{N}k} = T_s \sum_{n=bN-N}^{bN-1} \sum_{m=0}^{M-1} \xi_n^m e^{j2\pi m F n T_s} e^{-j\frac{2\pi n}{N}k}, \tag{9}$$

for $k = 0, \dots N-1, \; b = 1, \dots T$. The interferers $x^m, \; m = 0, 1, \dots M-1$ are, as previously stated, assumed to be mutually independent in the frequency domain. Assuming the interference is constant during one batch we have

$$X^m(kF_s, b) \;=\; T_s A_b^m \sum_{n=bN-N}^{bN-1} e^{j2\pi m F n T_s} e^{-j\frac{2\pi n}{N}k} = T_s A_b^m \sum_{n=bN-N}^{bN-1} e^{j\frac{2\pi n}{N}\left(\frac{mF}{F_s}-k\right)}, \tag{10}$$

$$X(kF_s, b) \;=\; \sum_{m=0}^{M-1} X^m(kF_s, b) \tag{11}$$

for $k = 0, \dots N-1, \; b = 1, \dots T$. By choosing the sampling period T_s and the number of data points N per batch, such that $F_s = F$ and $N = M$, we have the following result [18]

$$X^m(kF_s, b) \;=\; \begin{cases} NT_s A_b^m, & \text{if } m = k \\ 0, & \text{otherwise} \end{cases} \quad k = 0, \dots N-1, \; b = 1, \dots T. \tag{12}$$

The DFT of the observation noise is given by

$$W(kF_s, b) \;=\; T_s \sum_{n=bN-N}^{bN-1} w_n e^{-j\frac{2\pi n}{N}k}, \; k = 0, \dots N-1, \; b = 1, \dots T \tag{13}$$

and finally, the DFT of the spread spectrum signal is

$$S(kF_s, b) \;=\; T_s \sum_{n=bN-N}^{bN-1} s_n e^{-j\frac{2\pi n}{N}k}, \; k = 0, \dots N-1, \; b = 1, \dots T. \tag{14}$$

It is straightforward to show that $W(kF_s, b)$ and $S(kF_s, b)$ are zero mean iid random variables with variance $T_s^2 N \sigma_w^2$ and $T_s^2 N \sigma_s^2$, respectively, and that $W(kF_s, b)$ is Gaussian distributed. Invoking the central limit theorem (CLT), $S(kF_s, b)$ is approximately Gaussian distributed for large N. We then observe that the signals at each frequency bin k, consist of 1) a finite-state Markov chain $X(kF_s, b)$ and 2) for large N, additive white Gaussian noise. Hidden Markov theory can thus be used to extract the Markov chains impeded in the additive background noise. In the following two subsections, we use off-line and on-line maximum likelihood (ML) techniques for estimating and subtracting the interference.

Notation: For notational convenience, we choose to denote $Y(mF_s, b)$ at the frequency mF as Y_b^m. We denote the set $\{Y_b^m,\ b = 1, \ldots T\}$ as $\mathcal{Y}^m$ and $\{Y_b^m,\ b = b_1, \ldots b_2\}$ as $\mathcal{Y}_{b_1:b_2}^m$. Similar notation holds for $S(mF_s, b), X(mF_s, b)$ and $W(mF_s, b)$. Furthermore, we let $\mathcal{A}^m$ denote $\{A_b^m,\ b = 1, \ldots T\}$. Let $\theta^m \triangleq \{q_2^m, \Pi^m\},\ m = 0, \ldots M-1$.

3.1 Off-line Interference Suppression Scheme

We use a bank of M *independent* HMM smoothers in the frequency domain, that estimate, detect and subtract the interference present at each frequency bin. The estimation and detection of the mth, $m = 0, 1, \ldots M-1$, interferer is presented here. In order to simplify the notation for the reader, we here drop the superscript 'm' indicating the mth interferer.

Interference Parameter Estimation

The EM algorithm proposed in [19] is used to obtain the maximum likelihood (ML) estimate of θ which is denoted here as θ^{ML}. As a by-product of the E-step, conditional mean estimates of the state A of the mth interference is obtained. It is shown in [20] that under mild regularity conditions, the sequence $\left\{\theta^{(l)}\right\}$ for $l = 1, 2, \ldots$, of the EM algorithm converges to a stationary value of the likelihood function. The superscript '(l)' indicates the iteration number of the EM algorithm. Some of the results presented in this section are derived in [21]. The EM algorithm is outlined in Fig. 2.

First, the lth iteration of the E-step in the above mentioned EM algorithm is performed.

The Expectation Step:

The aim in the E-step is to evaluate $Q\left(\theta,\theta^{(l)}\right)$, defined in (47).

The probability density function for the complete data $(\mathcal{Y},\mathcal{A})$ can be expressed as

$$\ln f(\mathcal{Y},\mathcal{A}|\theta) = \sum_{b=1}^{T}\ln f(Y_b \mid A_b,\theta) + \ln f(A_1 \mid \theta) + \sum_{b=2}^{T}\ln f(A_b \mid A_{b-1},\theta) \tag{15}$$

where

$$f(Y_b \mid A_b,\theta) = \frac{1}{2\pi\sigma^2}e^{-\frac{(Y_b - T_s N A_b)^2}{\sigma^2}} \tag{16}$$

$$f(A_1 \mid \theta) = \prod_{i=1}^{2}\bar{\pi}^{\delta(A_1-q_i)} \tag{17}$$

$$f(A_b \mid A_{b-1},\theta) = \prod_{i,j=1}^{2}\pi_{ij}^{\delta(A_b-q_j)\delta(A_{b-1}-q_i)} \tag{18}$$

where $\delta(\cdot)$ denotes the Dirac delta function and σ^2 is the sum of the variance of the spread-spectrum signal and the variance of the observation noise after the DFT. Using Eq. (15), we compute the function defined in (47) to be

$$\begin{aligned} Q(\theta,\theta^{(l)}) &= E\left\{\ln f(\mathcal{Y},\mathcal{A}) \mid \mathcal{Y},\theta^{(l)}\right\} \\ &= -T2\pi\sigma^2 - \frac{1}{\sigma^2}\sum_{b=1}^{T}\left((Y_b)^2\gamma_b^{(l)}(1) + (Y_b - T_s N q_2)^2\gamma_b^{(l)}(2)\right) \\ &\quad + \sum_{i=1}^{2}\ln(\bar{\pi}_i)\gamma_b^{(l)}(i) + \sum_{b=2}^{T}\sum_{i,j=1}^{2}\ln(\pi_{ij})\gamma_b^{(l)}(i,j), \end{aligned} \tag{19}$$

where

$$\gamma_b^{(l)}(i) \triangleq \mathrm{P}\left(A_b = i|\mathcal{Y},\theta^{(l)}\right), \quad \gamma_b^{(l)}(i,j) \triangleq \mathrm{P}\left(A_b = j, A_{b-1} = i|\mathcal{Y},\theta^{(l)}\right) \tag{20}$$

for $b = 1,2,\ldots T,\ i,j \in \{1,2\}$.

From [21] the forward un-normalized a posteriori state probabilities, α, are recursively computed as follows

$$\begin{aligned} \alpha_b^{(l)}(j) &\triangleq f\left(Y_b, A_b = j|\theta^{(l)}\right) = \sum_{i=1}^{2} f\left(Y_b, A_b = j, A_{b-1} = i|\theta^{(l)}\right) \\ &= \sum_{i=1}^{2} f\left(Y_b|A_b = j,\theta^{(l)}\right)\pi_{ij}^{(l)}\alpha_{b-1}^{(l)}(i) \end{aligned} \tag{21}$$

and the backwards un-normalized probabilities, β, are given by

$$\begin{aligned}
\beta_b^{(l)}(i) &\triangleq f\left(\mathcal{Y}_{b+1:T}|A_b = i, Y_b, \theta^{(l)}\right) \\
&= \sum_{j=1}^{2} f\left(\mathcal{Y}_{b+2:T}, Y_{b+1}, A_{b+1} = j|A_b = i, Y_b, \theta^{(l)}\right) \\
&= \sum_{j=1}^{2} f\left(\mathcal{Y}_{b+2:T}|Y_{b+1}, A_{b+1} = j, \theta^{(l)}\right) f\left(Y_{b+1}|A_{b+1} = j, \theta^{(l)}\right) \pi_{ij}^{(l)} \\
&= \sum_{j=1}^{2} \beta_{b+1}^{(l)}(j) f\left(Y_{b+1}|A_{b+1} = j, \theta^{(l)}\right) \pi_{ij}^{(l)}
\end{aligned} \tag{22}$$

for $b = 1, 2, \ldots T,\ i, j \in \{1, 2\}$.

Initialization for the backward probabilities is $\beta_T^{(l)}(i) = 1,\ i \in \{1, 2\}$. The initial forward probabilities are given by $\alpha_1^{(l)}(i) = \bar{\pi}^{(l)} f(Y_1 \mid A_b = i, \theta^{(l)})$, for $i \in \{1, 2\}$.

Using the relations above the following relationships are achieved

$$\gamma_b^{(l)}(i) = \frac{\alpha_b^{(l)}(i)\beta_b^{(l)}(i)}{\sum_{i=1}^{2} \alpha_b^{(l)}(i)\beta_b^{(l)}(i)}. \tag{23}$$

and

$$\gamma_b^{(l)}(i, j) = \frac{\alpha_{b-1}^{(l)}(i) f\left(Y_b|A_b = q_j, \theta^{(l)}\right) \pi_{ij}^{(l)} \beta_b^{(l)}(j)}{\sum_{i,j=1}^{2} \alpha_{b-1}^{(l)}(i)\pi_{ij}^{(l)} f\left(Y_b|A_b = q_j, \theta^{(l)}\right) \beta_b^{(l)}(j)} \tag{24}$$

for $b = 1, 2, \ldots T,\ i, j \in \{1, 2\}$.

Note: When the model parameters are known, the a posteriori probabilities are computed via Equations (21), (22) and (23) by replacing the lth parameter estimates $\theta^{(l)}$ by the true parameters θ.

The Maximization Step:

The aim in the M-step is to update the parameter estimates $\theta^{(l+1)}$ from the previous estimates $\theta^{(l)}$ as follows

$$\theta^{(l+1)} = \arg\max_{\theta} Q(\theta, \theta^{(l)}). \tag{25}$$

I. The transition probabilities, π.

The following problem is to be solved

$$\begin{cases} \arg\max_{\pi_{ij}} Q(\theta, \theta^{(l)}) \\ \text{subject to :} \begin{cases} \sum_{j=1}^{2} \pi_{ij} = 1 \\ \pi_{ij} \geq 0 \qquad i, j = 1, 2 \end{cases} \end{cases}$$

The solution is as follows

$$\pi_{ij}^{(l)} = \frac{\sum_{b=1}^{T} \gamma_{b-1}^{(l)}(i,j)}{\sum_{b=1}^{T} \gamma_b^{(l)}(i)}, \qquad i,j \in \{1,2\}. \tag{26}$$

II. The amplitude level, q_2.

The state level q_2 on the $l+1$th iteration is computed by solving the following problem

$$\arg\max_{q_2} Q(\theta, \theta^{(l)}). \tag{27}$$

The solution is given by

$$q_2^{(l)} = \frac{\sum_{b=1}^{T} Y_b \gamma_b^{(l)}(2)}{\sum_{b=1}^{T} T_s N \gamma_b^{(l)}(2)}. \tag{28}$$

Detection (State Estimation)

Here, the goal is to detect the interferer's possible presence. Each interferer's amplitude is, as discussed previously, modeled as a 2-state Markov chain with one state-level being equal to zero and the other a non-zero complex number. When the state of the Markov chain is zero, the corresponding interferer is absent. Thus, by estimating the state of the mth interferer's Markov chain, we can conclude whether the mth interferer is present or not. The minimum probability of error estimate of detection of the interferer is given as follows

$$\begin{array}{ll} \text{interferer present if} & \gamma_b^{(l)}(2) \geq 1/2 \\ \text{interferer not present if} & \gamma_b^{(l)}(2) < 1/2 \end{array} \tag{29}$$

Interference Suppression

The outputs of the HMM smoother, $\gamma^{(l)}, q_2^{(l)}$, are used to estimate the mth interferer. The estimate of the narrowband interference at frequency mF_s on the bth data batch is given by the conditional mean estimate

$$\hat{X}_b^{(l)} \triangleq \mathrm{E}\left\{A_b | \mathcal{Y}, \theta^{(l)}\right\}, \tag{30}$$

where $\hat{X}_b^{(l)}$ denotes the estimate of X_b^m on the lth iteration. (30) reduces to

$$\hat{X}_b^{(l)} = \gamma_b^{(l)}(2) q_2^{(l)}, \ b = 1, \ldots T. \tag{31}$$

Equation (31) represents the estimate of the narrowband interferer at the mth frequency bin, which is computed by weighting the estimate of the interferer's signal strength by the probability that the interferer is active. The interference suppression is performed by subtracting the estimated interferer $\hat{X}_b$ from the data Y_b of the mth frequency bin, prior to taking the inverse discrete Fourier transform (IDFT). The spread spectrum signal after interference suppression is given by

$$\hat{s}_k = \frac{1}{NT_s} \sum_{m=0}^{N-1} \left(Y^m_{\mathrm{int}(k/N)} - \hat{X}^{m,(l)}_{\mathrm{int}(k/N)} \right) e^{j\frac{2\pi}{N}mk}, \; k = 0, 1, \ldots TN - 1, \tag{32}$$

where $\mathrm{int}(k/N)$ denotes the integer part of the division k/N, and $\hat{X}^{m,(l)}$ denotes the estimate of the interferer in the mth frequency bin computed on the lth iteration of the EM algorithm.

3.2 On-line Interference Suppression Scheme

We use a bank of M *independent* HMM filters in the frequency domain, that estimate, detect and subtract the interference present at each frequency bin. The estimation and detection of the mth interferer is presented below.

Interference Parameter Estimation

The on-line stochastic gradient algorithm studied in [22, 23] is applied to the posed problem. The model parameters are updated according to the following iterative scheme

$$\theta_b = \theta_{b-1} + \frac{1-\rho}{1-\rho^{b-1}} K_b I_b, \tag{33}$$

where ρ is a predetermined real constant and

$$I_b = \rho I_{b-1} + V_b\left(\theta_{b-1}\right), \; I_1 = V_1\left(\theta_0\right), \; I_0 = 0. \tag{34}$$

and θ_0 is the initial model parameter estimate. The incremental score vector, V_b, is defined by

$$V_b\left(\theta_{b-1}\right) \triangleq \left. \frac{\partial \mathrm{E}\left\{ \ln f\left(Y_b, A_b | \mathcal{Y}_{b-1}, \mathcal{A}_{b-1}, \theta\right)\right\}}{\partial \theta} \right|_{\theta=\theta_{b-1}}. \tag{35}$$

K_b denotes the step size at the bth time instant and is chosen as

$$K_b = \frac{1}{b}. \tag{36}$$

In order to speed the convergence, K_b could instead be chosen as b^{-l}, where l is a positive, real constant [24].

I. The transition probabilities, π.

By using a differential geometric approach suggested in [25], the formulae for updating the transition probabilities are given by the following set of equations

$$\pi_{ij,b} = \frac{g_{ij,b}}{\sum_{i,j=1}^{2} \left(g_{ij,b}\right)^2}, \tag{37}$$

where $\pi_{ij,b}$ denotes the transition probability estimate on the bth iteration, and

$$g_{ij,b} = g_{ij,b-1} + \frac{1-\rho}{1-\rho^{b-1}} K_b I_b, \quad I_b = \rho I_{b-1} + V_b\left(g_{ij,b-1}\right), \tag{38}$$

and

$$V_b\left(g_{ij,b-1}\right) = 2\left(\frac{\alpha_{b-1}(i,j)}{g_{ij,b-1}} - \alpha_{b-1}(i) g_{ij,b-1}\right). \tag{39}$$

II. The amplitude level, q_2.

The element of the incremental score vector corresponding to the state level q_2, is given by

$$V_b\left(q_{2,b-1}\right) = \left(Y_b - NT_s q_{2,b-1}\right) \frac{NT_s}{\sigma^2} \alpha_b(2), \tag{40}$$

where $q_{2,b}$ denotes the signal amplitude estimate of the mth interferer on the bth iteration. $\alpha_b(2)$ denotes the un-normalized a posteriori probability that the interferer is present on the bth iteration. $\alpha_b(2)$ is iteratively computed from the iteration in (21), by replacing the model parameters on the bth iteration by the model parameter estimates θ_b.

Detection (State Estimation)

The minimum probability of error in detecting the mth interferer using data up to time index b is given by

$$\begin{array}{ll} \text{interferer present if} & \alpha_b^{(l)}(2) \geq 1/2 \\ \text{interferer not present if} & \alpha_b^{(l)}(2) < 1/2 \end{array} \tag{41}$$

Interference Suppression

The outputs of the HMM filter, α, q_2, are used to estimate the mth interferer. The estimate of the narrowband interference at frequency mF_s on the bth data batch is given by the conditional mean estimate

$$\hat{X}_b \triangleq \mathrm{E}\left\{A_b | \mathcal{Y}_b, \theta_b\right\} = q_{2,b}\alpha_b(2), \tag{42}$$

where $\hat{X}_b$ denotes the estimate of X_b.

Finally, prior to taking the IDFT, the data Y_b in each frequency bin is reduced by the interfering estimate $\hat{X}_b$, that is

$$Y_b - q_{2,b}\alpha_b(2) \tag{43}$$

Computational Complexity

As stated previously, there is a major advantage to perform the detection, estimation and suppression of the interference in the frequency domain instead of in the time domain. If the detection of the interferers would have been carried out in the time-domain (see Eq.(4–5)), the optimal detector given our model assumptions would have been an HMM filter/smoother. The computational complexity of the time-domain HMM filter/smoother grows exponentially with the number of interferers. This is the case since we have to investigate all 2^M possible combinations of active and inactive interferers. Thus, our schemes have computational complexities that are linear in the number of interferers instead of exponentially growing as would have been the case if we would have worked in the time domain.

In Table 1, the computational complexities of our off-line and on-line interference schemes are given. Here, the complexity requirements for the Fourier transformation is not included. We choose to perform the transformation using the fast Fourier transform (FFT) which has a computational complexity of $\frac{N}{2}\log_2(N)$ per batch. Furthermore, the complexity requirements for subtracting the interference for both the on-line and off-line case is $O(M)$ per batch.

4 Experimental Results

In this section, we illustrate the performance of our proposed off-line and on-line schemes by way of simulation studies. We investigated the performance of our algorithms by varying the level of the amplitudes of the interfering signals, by varying the number of users transmitting spread spectrum signals, and finally by varying the number of possible narrowband interferers. The off-line and on-line schemes were evaluated for the case when the state level q_2^m is known and the case when the interference amplitude is unknown and has to be estimated. We chose to compare our schemes in terms of performance with the scheme suggested in [17]. That is, as a comparison we ran a bank of least mean squares (LMS) algorithms in the frequency domain, that replaces our bank of HMM smoothers/filters. The LMS algorithm for the mth interferer is given by

$$x_{b+1}^m = x_b^m + \mu(Y_b^m - x_b^m), \tag{44}$$

where $\mu \geq 0$. Suppression was computed by subtracting x_b^m from Y_b^m prior to taking the IDFT. Note that Eq. (44) reduces to our proposed online interference estimation scheme of (33) and (40) by setting $\rho = 0$, $K_b^m = \mu N T_s/\sigma^2$ and $\alpha_b^m(2) = 1$.

We evaluated the results in the time-domain and we used the SNR improvement as the performance measure. The SNR improvement is defined as

$$SNR_{imp} \triangleq \frac{E(|y_k - s_k|^2)}{E(|y_k - s_k - \hat{x}_k|^2)}, \tag{45}$$

where $\hat{x}_k$ denotes the estimate of the interference.

Common fixed parameters for all the simulations below were the following. We set the standard deviation of the additive observation noise to be $\sigma_w = 0.1$, the frequency $F = 30000$ and the transition probability matrix Π^m, was given by

$$\Pi^m = \begin{pmatrix} 0.99 & 0.01 \\ 0.01 & 0.99 \end{pmatrix}, \ \forall m \in \{1, \dots M\}. \tag{46}$$

The number of data batches was $T = 1600$. The update constant μ was chosen as 0.05. Fine-tuning of the on-line parameters was not thoroughly investigated. For all the simulation studies below, we ran 100 Monte Carlo simulations.

- *Varying the amplitudes, $q^m(2)$, of the interfering signals.*

The number of possible narrowband interferers were chosen as 16 and there was only one user transmitting a random sequence of ± 1. We varied the amplitudes $q^m(2), \ m = 0, \dots M-1$ of the interferers between 0.6 and 1.6. The amplitudes $q^m(2)$ for different $m \in \{0, \dots M-1\}$ were kept the same in each simulation. Firstly, we ran the off-line scheme with known state-levels $q_2^m, \ m = 0, \dots M-1$, and thereafter, we ran our off-line schemes assuming unknown state-levels $q_2^m, \ m = 0, \dots M-1$. The EM algorithm was iterated three times. The initialization of q_2^m was then set to a random non-zero number. Next, the on-line scheme assuming known state-levels $q_2^m, \ m = 0, \dots M-1$, was studied. Thereafter, the on-line scheme assuming unknown state-level $q_2^m, \ m = 0, \dots M-1$ was run. The initialization of q_2^m was set to a random non-zero number.

Finally, as a comparison a bank of LMS schemes was run under the same conditions. In Fig. 3, the results are presented.

- *Varying the number of users transmitting spread spectrum signals.*

The number of possible narrowband interferers were chosen as 16 and the amplitudes $q^m(2),\ m = 0, \dots M-1$ of the interferers were all chosen to $q^m(2) = 1,\ m = 0, \dots M-1$. The number of users transmitting random sequences of ± 1 was varied between 4 and 20. Firstly, we ran the off-line scheme with known state-levels $q_2^m,\ m = 0, \dots M-1$, and thereafter, we ran our off-line schemes assuming unknown state-levels $q_2^m,\ m = 0, \dots M-1$. The EM algorithm was iterated three times. The initialization of q_2^m was then set to a random non-zero number. Next, the on-line scheme assuming known state-levels $q_2^m,\ m = 0, \dots M-1$, was studied. Thereafter, the on-line scheme assuming unknown state-level $q_2^m,\ m = 0, \dots M-1$ was run. The initialization of q_2^m was set to a random non-zero number. Finally, as a comparison a bank of LMS schemes was run under the same conditions. In Fig. 4, the results are presented. Here, we notice a decrease in the performance as the number of users increase for the on-line scheme. Though, the degradation is not that severe for the off-line algorithm.

- *Varying the number of possible narrowband interferers.*

The number of users transmitting a random sequence of ± 1 was chosen to 1 and the amplitudes $q^m(2),\ m = 0, \dots M-1$ of the interferers were all chosen to $q^m(2) = 1,\ m = 0, \dots M-1$. The number of possible narrowband interferers was varied between 2 and 32. Firstly, we ran the off-line scheme with known state-levels $q_2^m,\ m = 0, \dots M-1$, and thereafter, we ran our off-line schemes assuming unknown state-levels $q_2^m,\ m = 0, \dots M-1$. The EM algorithm was iterated three times. The initialization of q_2^m was then set to a random non-zero number. Next, the on-line scheme assuming known state-levels $q_2^m,\ m = 0, \dots M-1$, was studied. Thereafter, the on-line scheme assuming unknown state-level $q_2^m,\ m = 0, \dots M-1$ was run. The initialization of q_2^m was set to a random non-zero number. Finally, as a comparison a bank of LMS schemes was run under the same conditions. In Fig. 5, the results are presented.

As can be seen from Figs. 3–5, the off-line scheme performs better than the on-line scheme, which is not surprising since off-line schemes smoothes the data while on-line scheme filters the data. To bridge the gap between the on-line and off-line schemes, fixed-lag smoothers can be used. The parameter estimation via the off-line scheme outperforms the parameter estimation of the on-line scheme. The major reason why the LMS algorithm does not perform as well is that its tracking of abrupt changes, *i.e.* detecting when interferers go on and off, is not as efficient as the HMM filter/smoother since the HMM estimator makes use of the knowledge of two discrete states. See Fig. 6 for a typical realization of the interfering signal amplitude and the estimation via the on-line scheme versus the LMS method.

5 Conclusions

In this paper, we have studied the suppression of multiple time-varying narrowband interferers in a spread-spectrum system and both off-line and on-line schemes have been proposed. The presence and statistics of the interferers are highly time-varying, *i.e.*, an interferer is allowed to appear, disappear and possibly be replaced by another interferer with different statistics. Our proposed algorithms detect, track and estimate these interferers by running a bank of hidden Markov model (HMM) estimators in the frequency domain. Each of the HMM estimators yields the conditional mean estimate of a narrowband interferer. The estimated narrowband signals are then removed prior to transforming back to time domain. The computational complexity of our proposed schemes is only linear in the number of interferers.

Future work will investigate the performance of our interference suppression schemes when the frequencies of the interferers are not equispaced. Other statistics of the interferers will be studied. Furthermore, the effects of multipath fading and the study of robustness to model mismatches will be addressed in future work.

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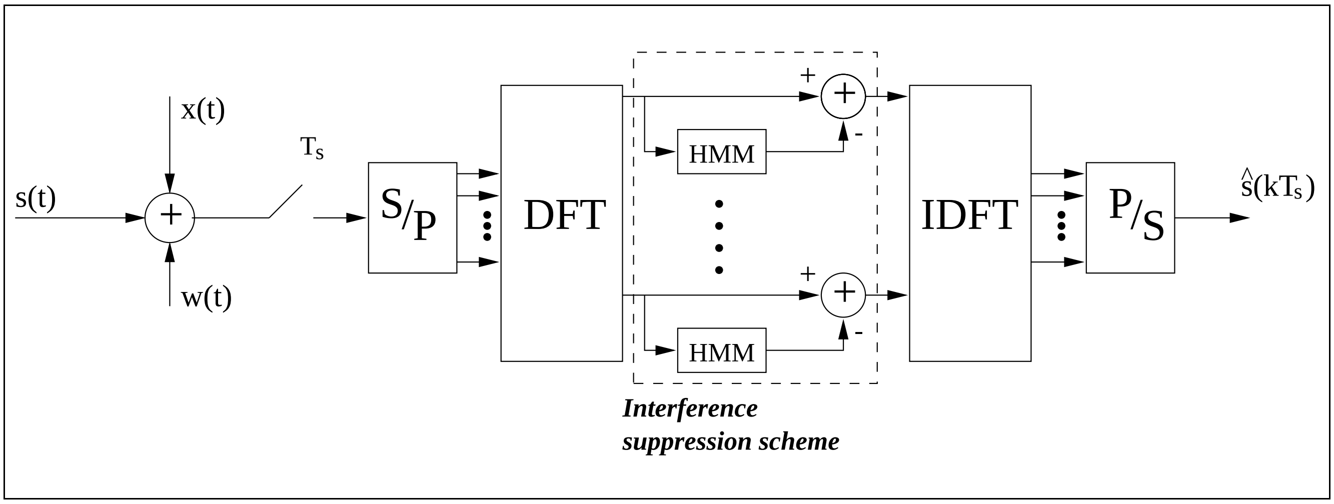

Figure 1: Schematics of the proposed interference suppression schemes using a bank of HMM estimators.

	Off-line complexity per pass	*On-line complexity per batch*
γ_b	$O(M2^2T)$	$O(M2^2)$
Π	$O(M2^2T)$	$O(M2^2)$
q_2	$O(MT)$	$O(M)$

Table 1: Computational complexity requirements of the algorithms.

Algorithm 1

0. *Determine the initial estimate* $\theta^{(1)}$.

1. Expectation step:
Evaluate

$$Q\left(\theta, \theta^{(l)}\right) \triangleq E\left\{\ln f(\mathcal{Y}, \mathcal{A} \mid \theta) | \mathcal{Y}, \theta^{(l)}\right\}. \tag{47}$$

2. Maximization step:
Compute

$$\theta^{(l+1)} = \arg\max_{\theta} Q\left(\theta, \theta^{(l)}\right). \tag{48}$$

3. $l := l + 1$. *Iterate steps 1–3 until* $\|\theta^{(l+1)} - \theta^{(l)}\| < \varepsilon$, *where* ε *is some specified constant.*

Figure 2: The EM algorithm

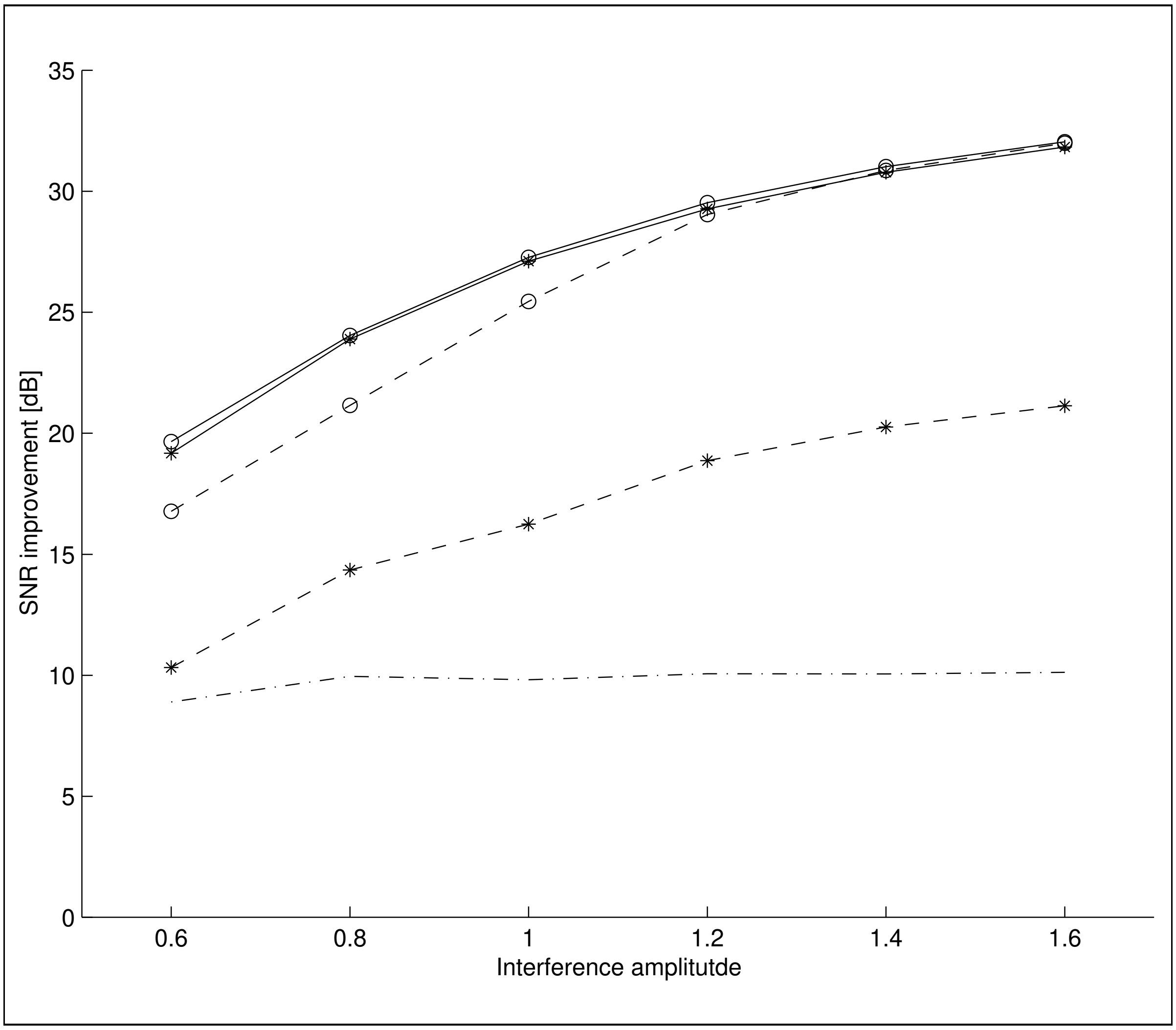

Figure 3: Performance in terms of SNR improvement for varying the amplitudes, $q^m(2)$, of the interfering signals. Off-line scheme results are denoted by solid lines '–', on-line scheme results by dashed lines '- -' and finally LMS algorithm results by dashed-dotted line '-.-'. We use 'o' and '*' to show the performance when the parameters are known and unknown, respectively.

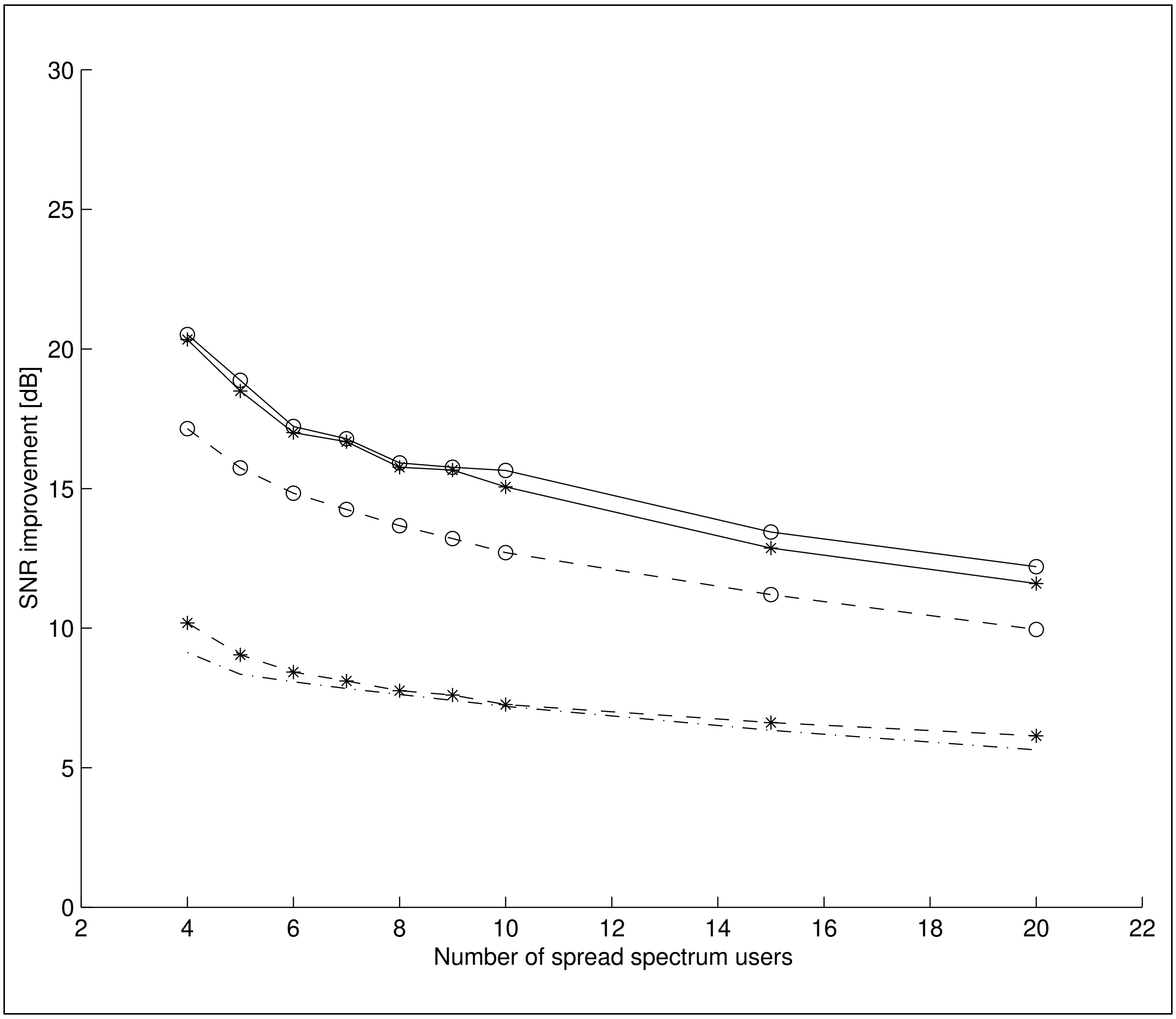

Figure 4: Performance in terms of SNR improvement for varying number of users transmitting spread spectrum signals. Off-line scheme results are denoted by solid lines '–', on-line scheme results by dashed lines'- -' and finally LMS algorithm results by dashed-dotted line '-.-'. We use 'o' and '*' to show the performance when the parameters are known and unknown, respectively.

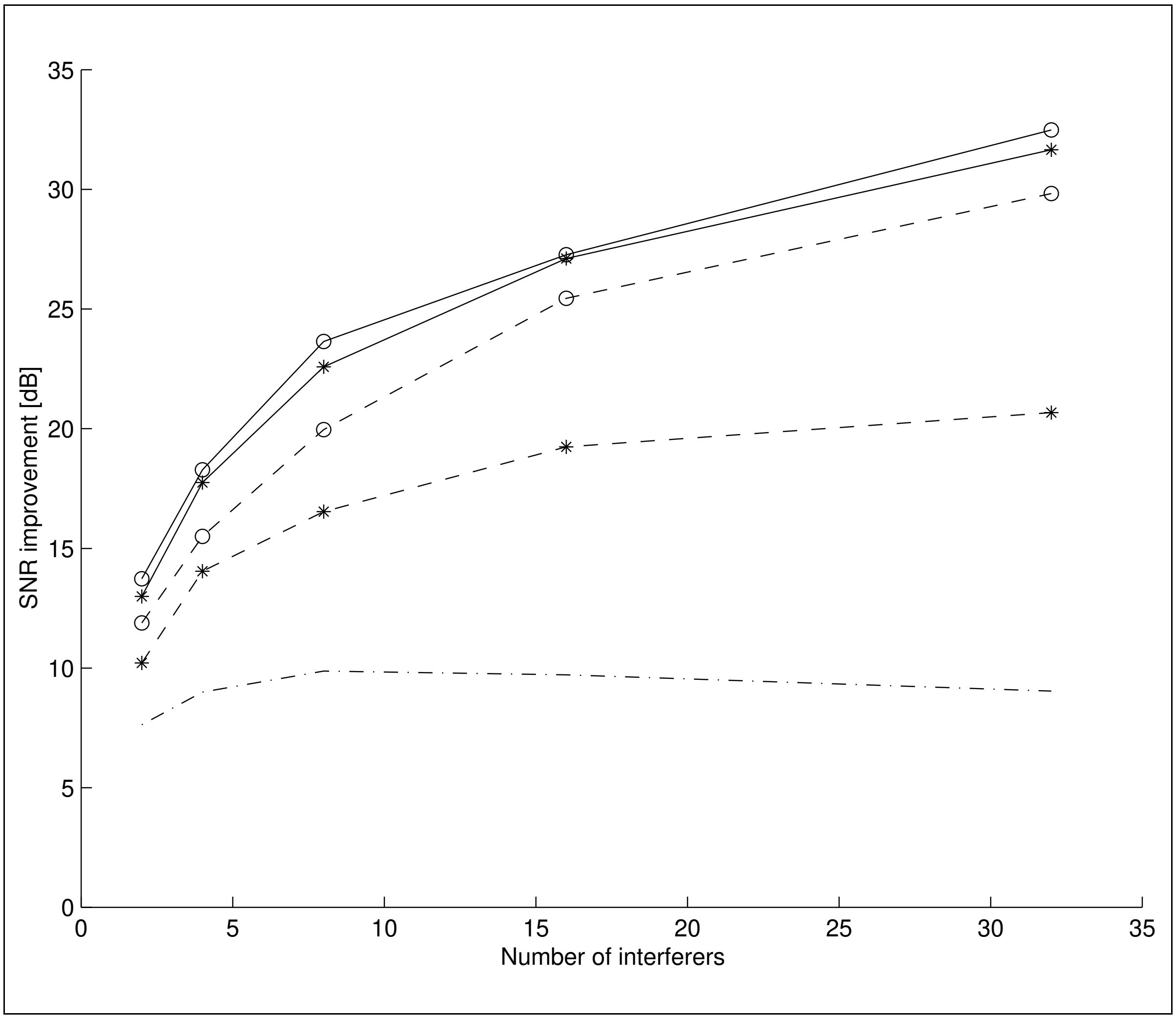

Figure 5: Performance in terms of SNR improvement for varying number of possible interferers. Off-line scheme results are denoted by solid lines '–', on-line scheme results by dashed lines '- -' and finally LMS algorithm results by dashed-dotted line '-.-'. We use 'o' and '*' to show the performance when the parameters are known and unknown, respectively.

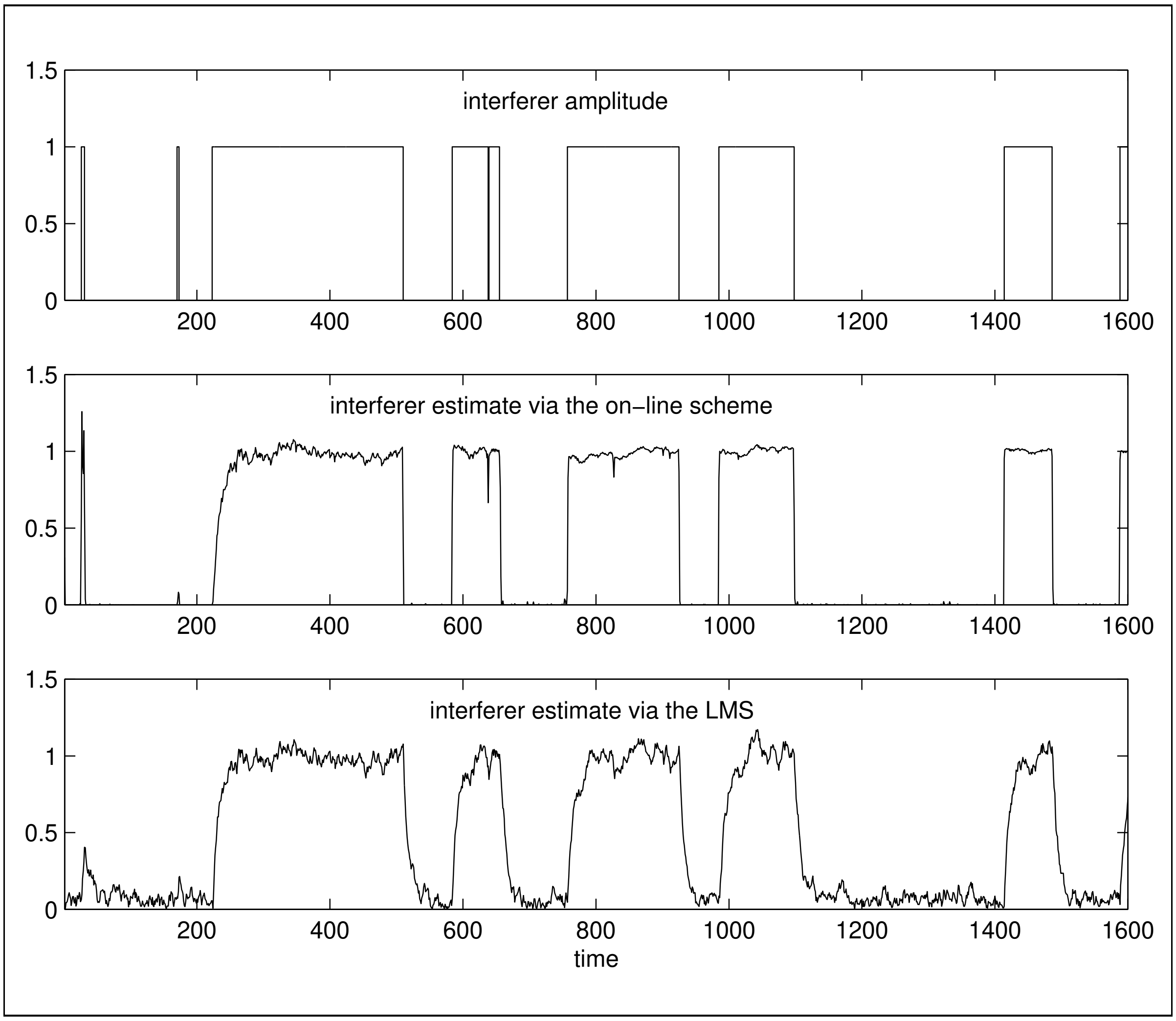

Figure 6: Typical tracking of the interference amplitude via the on-line scheme with parameter estimation versus the LMS algorithm.