A comparison of interference rejection and multiuser detection

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Abstract

We investigate the use of an antenna array at the receiver in FDMA/TDMA systems, to let several users share one communication channel within a cell. A decision feedback equalizer which simultaneously detects all incoming signals (multiuser detection) is compared to a set of decision feedback equalizers, each detecting one signal and rejecting the remaining as interference. We also introduce the existence of a zero-forcing solution to the equalization problem as an indicator of near-far resistance of different detector structures. Simulations indicate that multiuser detection in general provides better performance than interference rejection.

1 Introduction

In a cellular communication system, multi-element antennas, also known as *antenna arrays*, can be used at the receiver to increase the capacity. It may even be possible to employ *reuse within a cell*, i.e. to let several users share each channel in one cell. In this paper, we illustrate, compare and explore two ways of using an antenna array at the receiver to accomplish channel reuse within a cell:

1. Detect data from one user at a time while treating the other users as interference. In the following, this approach will be denoted *interference rejection* or *interference cancellation* [1, 2].
2. Detect the data from all users simultaneously. This will be called *multiuser detection* [3].

As will become evident in the following sections, the performance of (nonlinear) multiuser detectors is mostly superior to that of interference cancellers, for two reasons:

1. They can suppress interference more efficiently than non-linear interference cancellers.
2. The channel estimation is improved, which leads to more precise tuning of the detector.

We will illustrate the influence of these two factors by comparing two types of *decision feedback equalizers (DFEs)* in FDMA/TDMA systems:

1. The DFE presented in [2], which rejects interference.
2. The DFE of [4], which detects all signals simultaneously.

2 Channel models

The channel models upon which we base the design of the detectors are assumed to be linear, time-invariant and sampled at the symbol rate. We consider a case with M transmitters and N receivers[1]. For uplink transmission, the M transmitters are different mobiles in the cell, while the N receivers are located at the base station. In the downlink, we assume that a separate message is transmitted from each of M antenna elements at the base station. Each mobile is equipped with N receivers, which are used to detect one or several of the messages. In both links, we thus have M transmitters and N receivers. Assuming that $s_j(k)$ is the symbol sequence sent from transmitter j, while $x_i(k)$ is the signal at receiver i and $v_i(k)$ represents the additive noise, we define

$$s(k) \triangleq \begin{pmatrix} s_1(k) & s_2(k) & \dots & s_M(k) \end{pmatrix}^T \tag{1a}$$

$$x(k) \triangleq \begin{pmatrix} x_1(k) & x_2(k) & \dots & x_N(k) \end{pmatrix}^T \tag{1b}$$

$$v(k) \triangleq \begin{pmatrix} v_1(k) & v_2(k) & \dots & v_N(k) \end{pmatrix}^T \ . \tag{1c}$$

Furthermore, we define the scalar channel from transmitter j to receiver i

$$H_{ij}(q^{-1}) \triangleq H_{ij}^0 + H_{ij}^1 q^{-1} + \dots + H_{ij}^{L_{ij}} q^{-L_{ij}}$$

where H_{ij}^n are complex-valued constants and q^{-1} represents the unit delay operator. Finally, we introduce the polynomial matrix

$$\boldsymbol{H}(q^{-1}) \triangleq \begin{pmatrix} H_{11}(q^{-1}) & \dots & H_{1M}(q^{-1}) \\ \vdots & \ddots & \vdots \\ H_{N1}(q^{-1}) & \dots & H_{NM}(q^{-1}) \end{pmatrix} . \tag{2}$$

Using (1a), (1b), (1c) and (2), we can now express the signal received at the antenna array by the MIMO model

$$x(k) = \boldsymbol{H}(q^{-1})s(k) + v(k) \tag{3a}$$

$$= \mathbf{H}_0 s(k) + \dots + \mathbf{H}_L s(k-L) + v(k) \ . \tag{3b}$$

[1] The receivers can represent multiple antennas or polarization diversity branches. Fractionally spaced sampling is also a means of introducing several "equivalent" receivers. Finally, when the signal constellation is one-dimensional, the real and imaginary parts of the received baseband signals provide two measurements for each symbol to be detected, leading to a doubling of the equivalent number of receivers.

The multiuser detector is based on the model (3a). In (3b), L represents the maximum order of all scalar channels. The vector $v(k)$ of noise samples is characterized by the matrix-valued covariance function

$$\psi_{k-m} \triangleq E[v(k)v^H(m)] \tag{4}$$

and can be both spatially and temporally colored.

If we explicitly model the signal from only one of the users, we have to handle signals from the remaining users as interference. Assuming signal number 1 to be of interest, we define a disturbance vector $V(k)$ as the sum of all co-channel interference and noise:

$$V(k) = \sum_{n=2}^{M} \boldsymbol{H}_n(q^{-1})s_n(k) + v(k) \tag{5}$$

where $\boldsymbol{H}_n(q^{-1})$ is column n in (2). The interference $V(k)$ is characterized by its matrix valued covariance function

$$\bar{\psi}_{k-m} \triangleq E[V(k)V^H(m)] \ . \tag{6}$$

The interference rejection design is based on the model

$$x(k) = \boldsymbol{H}_1(q^{-1})s_1(k) + V(k) \ . \tag{7}$$

Remark 1. If we only model one of the signals explicitly, estimation of the matrix-valued covariance function of $V(k)$ is vital. This becomes a major problem, since direct estimation of $\bar{\psi}_m$ will provide poor accuracy for the short training sequences typically present in cellular systems. In fact, the estimates of the covariance function will be so unreliable that we in Subsection 4.2 are forced to exploit only the spatial structure of the covariance matrix, i.e. we will assume that $E[V(k)V^H(m)] = 0$ for $k \neq m$.

3 The multivariable DFE

3.1 The equalizer structure

A multivariable DFE with a transversal feedforward filter and a transversal feedback filter

$$\begin{aligned} \hat{s}(k-\ell|k) &= \boldsymbol{S}(q^{-1})x(k) - \boldsymbol{Q}(q^{-1})\tilde{s}(k-\ell-1) \\ \tilde{s}(k-\ell) &= \mathrm{f}(\hat{s}(k-\ell|k)) \end{aligned} \tag{8}$$

will be used. The output $x(k)$ of the array is used as input to the equalizer and $\tilde{s}(k-\ell-1)$ are the decisions previously made by the equalizer. The soft estimate $\hat{s}(k-\ell|k)$ is passed through the decision non-linearity $\mathrm{f}(\cdot)$ to produce the hard estimate $\tilde{s}(k-\ell)$. The feedforward filter $\boldsymbol{S}(q^{-1})$ is of order n_s, whereas the feedback filter $\boldsymbol{Q}(q^{-1})$ is of order n_Q. In general, (8) represents a MIMO DFE, which will be used for multiuser detection. With $\hat{s}$ scalar, i.e. for $M = 1$, we obtain a MISO DFE, which will be used for interference rejection. A set of M MISO DFEs can be represented by (8), with $\boldsymbol{Q}$ being *diagonal*. Note that the feedforward filter is causal, and that the decisions are made after a finite delay ℓ. This means that the DFE is always realizable, in contrast to the DFE presented in [1]. The use of FIR filters in (8) and of model-based (indirect) design of the equalizer is motivated in [5]. We adopt the common assumption that all previous decisions affecting the current symbol estimate are *correct*, i.e. that

$$\tilde{s}(k-\ell-n) = s(k-\ell-n) \quad n = 1,\ldots,n_Q+1 \ . \tag{9}$$

The coefficients of $\boldsymbol{S}(q^{-1})$ and $\boldsymbol{Q}(q^{-1})$ can be adjusted to obtain zero-forcing (ZF) or minimum mean-square error (MMSE) designs.

3.2 Zero-forcing and MMSE designs

A multiuser zero-forcing equalizer can be defined [1] as a filter which eliminates all intersymbol interference *and* co-channel interference:

Definition 1 *Consider the channel model* (3b) *and a multivariable equalizer which forms the estimate $\hat{s}(k-\ell|k)$ of a transmitted symbol vector $s(k-\ell)$. If*

$$\hat{s}(k-\ell|k) = s(k-\ell) + \varepsilon(k) \tag{10}$$

where $\varepsilon(k)$ is uncorrelated with all transmitted symbol vectors $s(m)$ $\forall$ m, then the equalizer is said to be zero-forcing.

By substituting (3a) and (9) into (8), the zero-forcing condition (10) implies:

$$\boldsymbol{S}(q^{-1})\boldsymbol{H}(q^{-1})s(k) - \boldsymbol{Q}(q^{-1})s(k-\ell-1) = s(k-\ell) \ .$$

A DFE will thus be zero-forcing if and only if $\boldsymbol{S}$ and $\boldsymbol{Q}$ constitute a solution to the *Diophantine equation*

$$\boldsymbol{S}(q^{-1})\boldsymbol{H}(q^{-1}) - q^{-\ell-1}\boldsymbol{Q}(q^{-1}) = q^{-\ell}\mathbf{I}_M \ . \tag{11}$$

The coefficients of an MMSE equalizer are determined to minimize

$$J = E[||s(k-\ell) - \hat{s}(k-\ell|k)||^2] \tag{12}$$

where the expectation is taken over the signal vector $s(k)$ in (1a) and the noise vector $v(k)$ in (1c) (MIMO case) or $V(k)$ in (5) (MISO case). Assuming (9), the design equations for the multivariable MMSE DFE can be obtained from two coupled Diophantine equations, which are transformed into a linear system of equations [5], [7].

Under conditions which will discussed in the next subsection, the MMSE DFE reduces to the ZF DFE when the noise variance goes to zero. In general, an MMSE equalizer provides better performance than a ZF equalizer, and in the simulation section, we will only utilize the MMSE DFE.

3.3 Near-far resistance, well-posedness and zero-forcing solutions

An MMSE DFE optimally balances suppression of intersymbol interference and co-channel interference against noise amplification. When the power of the interfering users is large, rejection of these strong signals is of paramount importance, whereas suppression of the noise is less important. This situation has been studied extensively for CDMA multiuser detectors, in which case the ability to cope with strong interferers is called *near-far resistance* [6].

We may then ask under what conditions are MIMO and MISO MMSE DFEs near-far resistant? To investigate this question, we let the noise covariance ψ_n tend to zero in (4) and in (6). If all intersymbol and co-channel interference can be removed, then the MMSE equalizer will reduce to a ZF equalizer, and the estimation error will vanish. In this case, perfect equalization is possible, for *any* power of the interfering users. If no ZF equalizer exists, all intersymbol and co-channel interference cannot be removed, so the estimation error will not vanish.

We can therefore use the existence of a ZF DFE as an indication of near-far resistance for the MMSE MIMO DFE or the MMSE MISO DFE. In more general terms, the existence of a zero-forcing solution also indicates that the equalization problem is well-posed in the sense that it can provide a useful solution: Good performance can be guaranteed, for sufficiently low noise levels.

Under mild conditions, DFEs which fulfill the zero-forcing condition (11) exist. However, it remains to specify the filter degrees of such DFEs. This is the topic of Theorem 1 below. As a prerequisite, we need the following definitions:

$d_j \triangleq$ the propagation delay of user j

$g_j \triangleq$ the degree of the greatest common polynomial factor in the column $\boldsymbol{H}_j(q^{-1})$ other than q^{-d_j}

$\bar{L}_j \triangleq \max_i L_{ij} - d_j - g_j$

We are now ready to formulate Theorem 1.

Theorem 1 *Consider the MIMO channel model* (3b) *with M sources and N sensors with $M \leq N$ and assume a zero-forcing solution to exist. A generically necessary condition for the existence of a zero-forcing MIMO DFE* (8) *with decision delay ℓ and feedforward filter degree n_s is then that*

$$n_s \geq \frac{M(\ell+1) - \sum_{m=1}^{M} d_m}{N} - 1 \,. \qquad (13)$$

The condition

$$n_s \geq \frac{\sum_{m=1}^{M} \bar{L}_m + \ell + 1 - \bar{L}_j - d_j}{N + 1 - M} - 1 \qquad (14)$$

is generically necessary for the existence of a set of M MISO DFEs with decision delay ℓ and feedforward filter degree n_s.

Proof: See [5]. ■

When either the condition (13) or the condition (14) is violated, the corresponding detector will not have enough degrees of freedom to completely cancel all the interfering signals.

The impact of a violation of the inequality (14) will be demonstrated in Subsection 4.1.2.

4 Monte Carlo simulations

The performance of the MIMO DFE is compared by simulation with the performance of the MISO DFE. The MIMO DFE jointly detects all the signals, whereas the MISO DFE only detects one of them. In both DFEs, the smoothing lag and feedforward filter lengths equal the length of the channel, $\ell = ns = L$.

In our scenario, one, two, three or four BPSK modulated signals impinge on an antenna array with four antenna elements. Each signal passes through a frequency selective, three-tap Rayleigh fading channel and the taps of the channels fade independently. The channels are time-invariant over the duration of a TDMA burst. The channels from different transmitters to a given receiver are independent, and so are the channels from any given transmitter to different receivers.[2] All signals are received in the presence of additive Gaussian noise, which is both spatially and temporally white.

The performance of the algorithms will be addressed as a function of the average SNR per bit [8]

$$\bar{\gamma}_b^j = \frac{1}{N} \frac{E[|H_{ij}^0|^2 + |H_{ij}^1|^2 + |H_{ij}^2|^2] E[|s_j(k)|^2]}{E[|v_i(k)|^2]} \,. \qquad (15)$$

We assume that $\bar{\gamma}_b^j$ is equal at different antenna elements and thus independent of i.

4.1 Known channels and noise covariances

In this subsection we shall study the idealized case when all channel coefficients are exactly known. Effects caused by differences in detector structure can here be studied in isolation, since effects of channel estimation errors are avoided.

4.1.1 Equal average SNR

We will here assume that all users have the same average SNR. This can be accomplished by using slow power control, which compensates for the propagation loss and shadow fading, but not for the fast fading.

Fig. 1 shows the estimated BER as a function of the average SNR per bit. With four users, the performance

[2] These assumptions may not always be valid. However, successful multiuser detection does not require uncorrelated channels to different receivers (uncorrelated antennas). See e.g. [5],[7] for a discussion of how antenna correlation affects the performance.

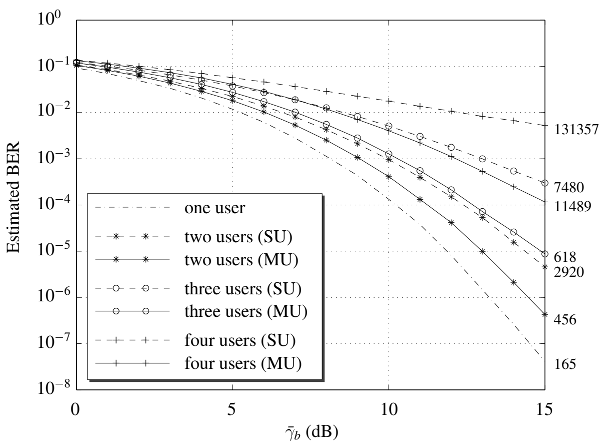

Figure 1: Comparison of the MIMO DFE (MU) and the MISO DFE (SU) for known channels. The numbers to the right of the graph are the number of errors used to estimate the BER for the average SNR per bit $\bar{\gamma}_b$=15 dB.

of the MIMO DFE at $\bar{\gamma}_b^j = 15$ dB is around 6 dB better than the performance of the MISO DFE. This difference arises from the fact that the MISO DFE uses up all its degrees of freedom to cancel the interference from the other users. This task is easier for the MIMO DFE since its feedback filter takes care of some of the interference suppression. For fewer users, the difference between the two approaches is smaller. For example, in the case of three users the gain is approximately 3 dB and for two users around 1 dB.

4.1.2 Unequal average SNR

In Subsection 4.1.1, we assumed that power control was used to compensate for the propagation loss and the shadow fading. In the scenario investigated in this subsection, we will relax this assumption: Even the average received powers will differ among the users. This will generate the so-called *near-far problem*.

We estimated the BER of a user having an average SNR per bit of 10 dB in a scenario where there are one, two or three additional users, each having an average SNR per bit that is between 0 dB and 10 dB *higher*, i.e. between 10 dB and 20 dB. The result from this simulation is depicted in the *right* half of Fig. 2.

In a MIMO DFE, decisions concerning one user affect future symbol estimates of all users. Incorrect decisions on the symbols from a weak user will thus impair the decisions of other, stronger users. In this case, a MISO DFE may yield better performance since (possibly incorrect) decisions of the weaker users' symbols do not influence the estimates the stronger users' symbols.

To investigate this effect, we estimate the BER of a user having an average SNR per bit of 10 dB in a scenario where there were one, two or three additional users, each having an average SNR per bit which was between 0 dB and 10 dB *lower*, i.e. the SNR per bit of the remaining users varied between 10 dB and 0 dB. The result from this simulation is depicted in the *left* half of Fig. 2.

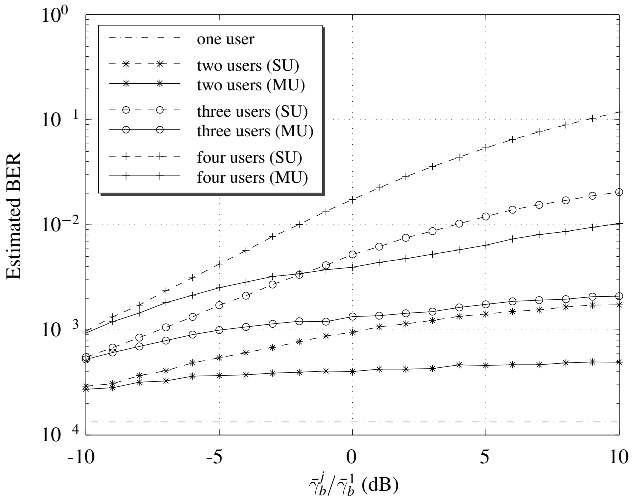

Figure 2: Comparison of the MIMO DFE (MU) and the MISO DFE (SU) for known channels and different transmitter powers. *In this simulation, 25000 channels were randomly selected. Over each channel, 1000 symbols were transmitted. User number 1 has an SNR per bit of $\bar{\gamma}_b^1 = 10$ dB, while the SNR per bit $\bar{\gamma}_b^j$ of the other users are equal and varies.*

From the leftmost part of Fig. 2, it is clear that for the investigated differences in power levels, error propagation is not so severe that the BER of a MIMO DFE exceeds the BER of a MISO DFE. On the other hand, from the rightmost part of Fig. 2, it is evident that for the MIMO DFE, four users can co-exist in the cell, even when the received average powers differ substantially. However, the performance of the MISO DFE is seriously affected by the increase of the power levels of the interfering users, since this MISO DFE does not comply with the ZF condition (14). Inserting numerical values into (14), we find that complete suppression of all co-channel interferers is impossible whenever $M \geq 3$. As the transmitter powers of these users increase, the estimation error due to residual interference increases, resulting in an increased BER. The MIMO DFE on the other hand is capable of completely removing the interference from the stronger users, at the expense of a slightly increased noise amplification.

4.2 Estimated channel coefficients

To demonstrate how the MIMO DFE works in a more realistic case, channel estimation is introduced. The data is transmitted in bursts, with a structure similar to that of GSM: A training sequence of 26 symbols is located in the middle of each burst. Together with data symbols, tail symbols and control symbols, this results in a total burst length of 148 symbols. The channel estimation is performed using the off-line least squares method, and the spatial color of the noise is estimated from the residuals of the channel identification. The temporal color of the noise is neglected due to the limited amount of data. Apart from this, the simulation conditions are the same as in Subsection 4.1.1 The results are indicated in Fig. 3.

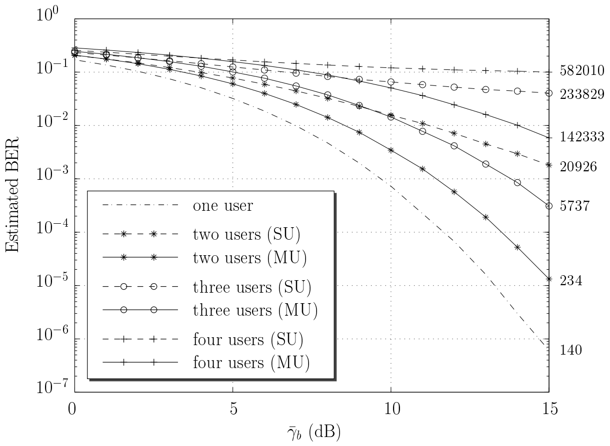

Figure 3: Comparison of the MIMO DFE (MU) and the MISO DFE (SU) for estimated channels.

When we compare Figs. 1 and 3, we see that the difference between the MIMO DFE and the MISO DFE is greater when the channels have to be estimated. The inability to estimate and subsequently use the temporal color of the interference leads to a larger performance degradation for interference rejection. Again, the difference in performance is larger with more active users.[3]

5 Discussion and conclusions

In our investigation of receiver algorithms designed to accomplish channel reuse within cells, MIMO DFEs which work as multiuser detectors have been compared to the use of interference rejection, implemented by MISO DFEs. In summary, simulations indicate that multiuser detection provides superior performance.

Differences in performance between multiuser detection and interference rejection are partly due to the detector *structures*: A multiuser (MIMO) DFE utilizes feedback from previously estimated symbols from *all* users, while the interference rejecting (MISO) DFE performs decision feedback from the user of interest only. The difference also results from the preconditions for *channel estimation*: In the multiuser case, input-output transfer functions can be estimated. For interference rejection, the co-channel interference constitutes colored noise. The multivariate noise models estimated from short data records will have poor accuracy.

Multiuser detectors and interference rejecting MISO DFEs can both be made near-far resistant. However, the conditions for this, as indicated by the existence of a zero-forcing solution, are somewhat more restrictive for interference rejection.

In [5], [7], the MIMO and MISO DFEs are applied to experimental measurements from an antenna array testbed. The results from this trial confirm the results presented in this paper and indicate that reuse within a cell is indeed possible, using either an eight element array antenna, or a conventional sector antenna with two polarization diversity branches.

[3]For cases with BER $< 10\%$, the BER can be reduced significantly both for the MIMO and MISO DFE by using a multi-pass "bootstrap" algorithm [9], where decisioned data are used to improve estimates of the channel and noise parameters [7].

Acknowledgements

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