Realizable MIMO Decision Feedback Equalizers

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Multivariable channel models

➢ Detection using antenna arrays

$$\begin{pmatrix} y_1(k) \\ \vdots \\ y_{n_y}(k) \end{pmatrix} = \begin{pmatrix} \boldsymbol{\mathcal{H}}_{11}(q^{-1}) & \dots & \boldsymbol{\mathcal{H}}_{1n_d}(q^{-1}) \\ \vdots & \ddots & \vdots \\ \boldsymbol{\mathcal{H}}_{n_y1}(q^{-1}) & \dots & \boldsymbol{\mathcal{H}}_{n_yn_d}(q^{-1}) \end{pmatrix} \begin{pmatrix} d_1(k) \\ \vdots \\ d_{n_d}(k) \end{pmatrix} + \begin{pmatrix} v_1(k) \\ \vdots \\ v_{n_y}(k) \end{pmatrix}$$

➢ Multiuser detection in DS-CDMA

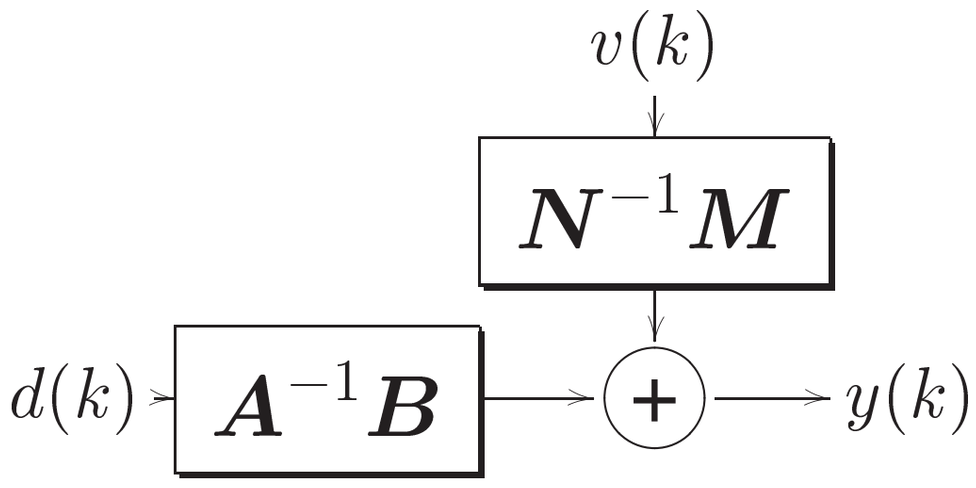
$$\begin{pmatrix} y_1(k) \\ \vdots \\ y_{n_y}(k) \end{pmatrix} = \mathbf{R}(1)\mathbf{W}\begin{pmatrix} d_1(k) \\ \vdots \\ d_{n_d}(k) \end{pmatrix} + \mathbf{R}(0)\mathbf{W}\begin{pmatrix} d_1(k-1) \\ \vdots \\ d_{n_d}(k-1) \end{pmatrix} + \mathbf{R}(-1)\mathbf{W}\begin{pmatrix} d_1(k-2) \\ \vdots \\ d_{n_d}(k-2) \end{pmatrix} + \begin{pmatrix} v_1(k) \\ \vdots \\ v_{n_y}(k) \end{pmatrix}$$

where the matrices $\mathbf{R}(m)$ contain partial cross-correlations between spreading sequences:

$$(\mathbf{R}(m))_{i,j} \stackrel{\triangle}{=} \int_{\tau_i}^{\tau_i+T_s} s_i(t-\tau_i)s^*(t+mT_s-\tau_j)dt$$

and $\mathbf{W}$ is a diagonal matrix containing the transmitter powers.

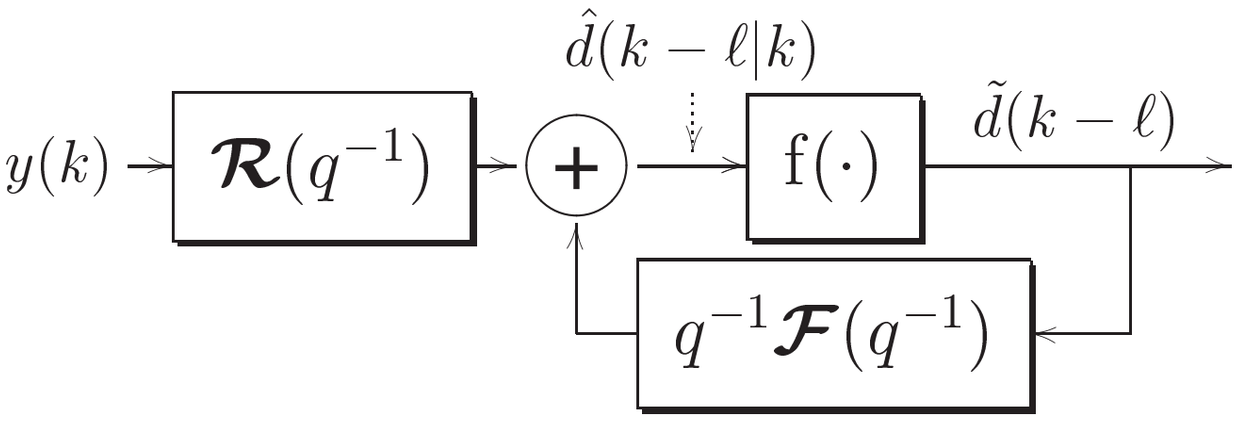
A general channel model structure



$$y(k) = \boldsymbol{A}^{-1}\boldsymbol{B}d(k) + \boldsymbol{N}^{-1}\boldsymbol{M}v(k)$$

$\det \boldsymbol{P}(z^{-1}) = 0$ has all roots in $|z| \leq 1,\ \boldsymbol{P} = \{\boldsymbol{A}, \boldsymbol{M}, \boldsymbol{N}\}$

A general DFE



$$\hat{d}(k-\ell|k) = \boldsymbol{\mathcal{R}}y(k) - \boldsymbol{\mathcal{F}}\tilde{d}(k-\ell-1)$$
$$\tilde{d}(k-\ell) = \mathrm{f}(\hat{d}(k-\ell|k))$$

Two criteria used for parameter adjustment:

Minimum mean-square error: Under the constraint of realizability of $\{\boldsymbol{\mathcal{R}}, \boldsymbol{\mathcal{F}}\}$, minimize

$$\mathbf{P} \stackrel{\triangle}{=} E\varepsilon(k-\ell)\varepsilon^H(k-\ell)$$

$$\varepsilon(k-\ell) \stackrel{\triangle}{=} d(k-\ell) - \hat{d}(k-\ell|k)$$

Zero-forcing: $\hat{d}(k-\ell|k) = d(k-\ell) - \varepsilon(k-\ell)$

$\varepsilon(k-\ell)$ uncorrelated with $d(m)\ \forall m$

MMSE design equations

➢ Let

$$\boldsymbol{\Gamma} \stackrel{\triangle}{=} \boldsymbol{A}\boldsymbol{M}$$
$$\boldsymbol{\tau} \stackrel{\triangle}{=} \boldsymbol{N}\boldsymbol{B}$$

and

$$\tilde{\boldsymbol{\tau}}\tilde{\boldsymbol{\Gamma}}^{-1} \stackrel{\triangle}{=} \boldsymbol{\Gamma}^{-1}\boldsymbol{\tau}$$

➢ Then the MMSE optimal general DFE (GDFE) is obtained as

$$\boldsymbol{\mathcal{R}} = \boldsymbol{S}\ \boldsymbol{M}^{-1}\boldsymbol{N} \qquad \boldsymbol{\mathcal{F}} = \boldsymbol{Q}\tilde{\boldsymbol{\Gamma}}^{-1}$$

where

$$\tilde{\boldsymbol{\Gamma}} - q^{\ell}\boldsymbol{S}\tilde{\boldsymbol{\tau}} + q^{-1}\boldsymbol{Q} = \boldsymbol{L}_{1*}\tilde{\boldsymbol{\Gamma}}$$
$$q^{-\ell}\boldsymbol{L}_{1*}\boldsymbol{\tau}_* - \rho\boldsymbol{S}\boldsymbol{\Gamma}_* = q\boldsymbol{L}_{2*}$$

(Linear system of equations)

ZF design equations

➢ The zero-forcing criterion is satisfied iff

$$q^{-\ell}\mathbf{I} = \boldsymbol{\mathcal{R}}\boldsymbol{A}^{-1}\boldsymbol{B} - q^{-\ell-1}\boldsymbol{\mathcal{F}} \qquad (\star)$$

➢ A solution to $(\star)$ exists iff every common right factor of $\boldsymbol{A}^{-1}\boldsymbol{B}$ and $q^{-\ell-1}\mathbf{I}$ is also a right factor of $q^{-\ell}\mathbf{I}$

➢ Let $\Delta_i \stackrel{\triangle}{=}$ the minimal delay of user i in any channel. If

- $n_d > n_y$, or
- $\Delta_i > \ell$ for some i

then $\boldsymbol{A}^{-1}\boldsymbol{B}$ and $q^{-\ell-1}\mathbf{I}$ will always have a common factor which is *not* a factor of $q^{-\ell}\mathbf{I}$

➢ The presence of a solution to $(\star)$ can be used as an indication of near-far resistance.

A numerical example: Monte Carlo simulation

➢ Channel: $y(k) = \boldsymbol{B}(q^{-1})d(k) + \boldsymbol{M}(q^{-1})v(k)$ with

$$\boldsymbol{B}(q^{-1}) = \begin{pmatrix} 0.979 + 0.204q^{-1} & 0.826 + 0.563q^{-1} \\ -0.843 - 0.538q^{-1} & 0.403 + 0.915q^{-1} \end{pmatrix}$$

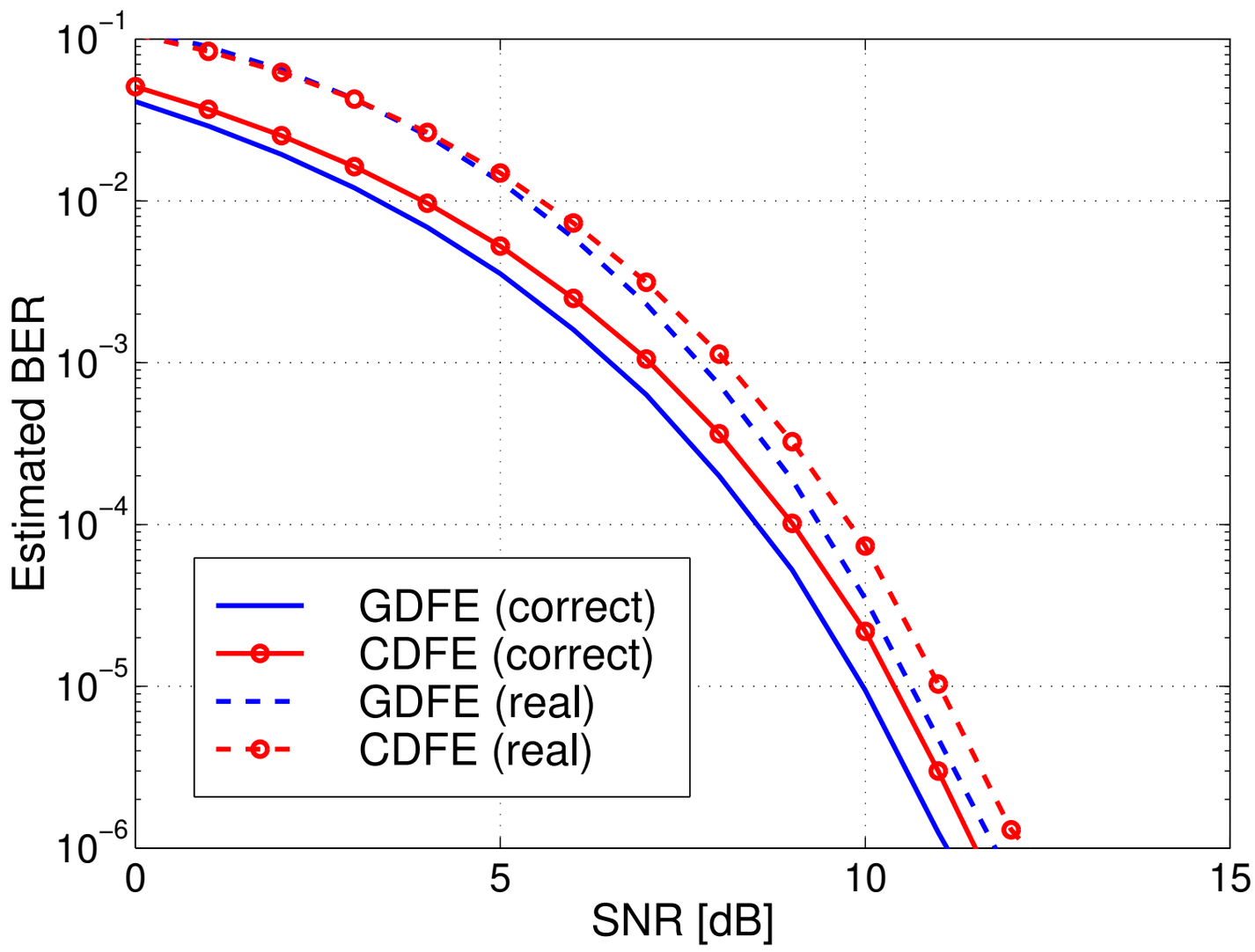
$$\boldsymbol{M}(q^{-1}) = \begin{pmatrix} -0.409 - 0.179q^{-1} & -0.535 + 0.717q^{-1} \\ -0.507 + 0.361q^{-1} & 0.761 - 0.181q^{-1} \end{pmatrix}$$

The roots of $\boldsymbol{M}(q^{-1})$ are located in $0.298 \pm 0.321i$.

➢ Comparison of

- ➪ GDFE
- ➪ Conventional DFE (CDFE)

➢ Binary data transmitted and the fraction of incorrect decisions from the decision device recorded



Conclusions

➢ Optimum structure of realizable MIMO Decision Feedback Equalizer derived

➢ Closed form design equations for MMSE and ZF DFE

➢ Near-far resistance coupled to existence of zero-forcing solution

➢ The GDFE performs significantly better than the conventinonal DFE when the SNR is high.