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MULTI-USER CHANNEL ESTIMATION EXPLOITING PULSE SHAPING INFORMATION

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ABSTRACT

A method for joint estimation of wireless communication channels to multiple users utilizing pulse shaping knowledge is presented. The pulse shaping in the transmitter and the receiver is incorporated into the channel estimation by approximating it with a set of pulse shaping functions. In joint multi-user channel estimation the number of parameters to be estimated grows linearly with the number of users while the number of equations remains constant. Utilizing the pulse shaping information is therefore especially useful since this economizes on the number of parameters to be estimated per user. This is illustrated with a multi-user channel estimation example.

1. INTRODUCTION

A wireless communication system is often interference limited, i.e. the signal quality is limited because of co-channel interference rather than because of noise. If the channels to the interfering signals can be estimated jointly with the desired signal, the channel estimates of the desired signal as well as of the interferers can be improved. This can be utilized in the symbol detection.

Since in joint multi-user channel estimation the number of parameters to be estimated grows linearly with the number of users while the number of equations remains constant, it is important to use as few parameters as possible per user.

One way to economize on the number of parameters to estimate is to utilize knowledge of the pulse shaping in the transmitter and the receiver and thus effectively only estimate the unknown propagation channel as in [1].

Some other methods exists for utilizing pulse shaping information for the single user case. One such method can be found in [2], where a where a GSM channel is estimated by cross-correlation of the received signal and the transmitted modulated signal. The training sequence is chosen so that this cross-correlation approximately will give the propagation part of the channel, i.e excluding the pulse shaping. Another method is presented in [3]. Here the continuous convolution between the pulse shaping function and the unknown remaining channel impulse response is discretized. This results in a parametrization of the total channel in terms of parameters for the unknown part of the channel. This parametrization is then used in order to identify the total channel.

Other approaches to utilizing pulse shaping information for channel estimation can for example be found in [4], and [5]. A blind method using pulse shaping information can also be found in [6].

The channel identification method presented here also derives a parametrization of the the total channel in terms of parameters for the unknown part of the channel, mainly consisting of the propagation channel. The approach to the modeling is however different. The method is based on approximation using a set of pulse shaping functions sampled at different time instances. This can approximately be viewed as an interpolation between sampled versions of the pulse shaping function with different offsets in the sampling instances, similar to the approach in [7]. A more detailed presentation of the method can be found in [1].

Since the method presented here does economize on the number of parameters to be estimated for each user, it will improve the channel estimates in line with the parsimony principle. In some cases it will also make otherwise impossible joint multi-user channel estimation possible. We illustrate this by studying a multi-user channel identification example.

2. CHANNEL MODEL

We start by modeling the channel for a single user. In continuous time, a linear communication channel with linear modulation can be modeled by a *known* linear pulse shaping filter, $\mathrm{p}(t)$, and an *unknown* linear filter, $\mathrm{h}(t)$, mainly representing the propagation channel the signal passes through. In the pulse shaping filter we can include all known linear filtering which is performed both at the receiver and the transmitter. The unknown linear filtering performed by the propagation channel and possible unknown filtering in the receiver and transmitter can be modeled by the channel, $h(t)$. This continuous time model can be seen in Figure 1.

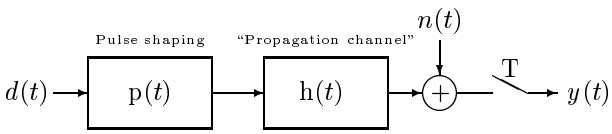

Figure 1: *Continuous time channel model.*

Since the input symbols, $d(t)$, $t = 1, 2, \ldots$, are discrete in time and the received signal, $y(t)$, is sampled at $t = 1, 2, \ldots$, the resulting discrete time channel can be modeled with an FIR filter, $\mathrm{B}(q^{-1}) = \mathrm{b}_0 + \mathrm{b}_1 q^{-1} +$

$\ldots + b_{nb}q^{-11}$, as in Figure 2. By using a known training sequence, the taps in $B(q^{-1})$ can be estimated with a simple least squares method. By doing this we have however neglected to make use of the knowledge of the pulse shaping filter $p(t)$.

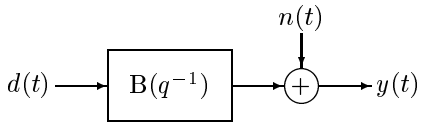

Figure 2: *Discrete FIR channel model.*

A first step towards incorporating the knowledge of the pulse shaping filter into the discrete channel model is to sample $p(t)$ T-spaced (symbol spaced) and form an FIR filter $P(q^{-1})$ from the samples. The discrete time model then becomes $P(q^{-1})$ followed by a T-spaced discretization, $H(q^{-1})$ of the channel $h(t)$. This model can be seen in Figure 3.

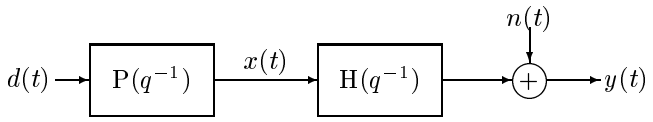

Figure 3: *Discrete channel model with a T-spaced sampled pulse shaping filter.*

The estimation is now restricted to the estimation of the FIR channel filter $H(q^{-1})$ using the "modulated" signal $x(t)$ as the input signal. A potential problem with this method is that the model may be to coarse. For a pulse passing through the system being sampled at time instances inbetween the chosen sampling points of $p(t)$, there will be a discrepancy in the model. The approximation of such a sampled pulse will essentially be a linear combination of two shifted versions of the sampled pulse shaping function, $P(q^{-1})$. Apart from a scaling this can be viewed as an interpolation between the two shifted pulse shaping filters. This will result in a representation with essentially two adjacent taps in the channel $H(q^{-1})$.

If improved accuracy is desired in the model, more than one sampled version of the pulse shaping function $p(t)$, similarly to [7], can be used in the approximation. Further we may chose to consider fractionally spaced sampling. A model including these two additions can be seen in Figure 4. We have here two fractionally spaced *sampling branches*, each with two *modeling branches*.

For each sampling branch, the modeling of the channel is divided up into two modeling branches with two different sampled versions of the pulse shaping filter, e.g. $P_{0.0}(q^{-1})$ and $P_{0.5}(q^{-1})$ for the upper sampling branch, with their sampling instances offset by half a symbol interval. The subscript refers to the offset of the filters center tap from the center or peak of the pulse shaping function $p(t)$. See the example in Figure 6.

Each discrete time pulse shaping filter is then followed by a discrete time channel filter, $H_1(q^{-1})$ or $H_2(q^{-1})$.

[1]We are here using the delay operator q^{-1}, $q^{-1}d(t) = d(t-1)$

Each pulse passing through the system will typically be approximated using a total of two taps in the filters $H_1(q^{-1})$ or $H_2(q^{-1})$. Again this can be viewed as an interpolation among adjacent sampled and shifted pulse shaping functions. Note that the interpolation now is performed among T/2-spaced sampled and shifted pulse shaping functions, $P_{0.0}(q^{-1})$ and $P_{0.5}(q^{-1})$. This interpolation is thus improved compared to the T-spaced interpolation in Figure 3. If an even finer model is desired, more pulse shaping filters with less spacing of the sampling instances can be used for the modeling of each sampling branch.

Fractionally spaced sampling has been introduced by adding an extra channel for the received signal sampled at offset values. Since the timing of the two sampling branches are offset by $0.5T$ the sampled pulse shaping functions will correspondingly be offset by $0.5T$ as seen in Figure 4.

Note that since both sampling branches share the same continuous time channel, $h(t)$, the same channel filters, $H_1(q^{-1})$ and $H_2(q^{-1})$, can be used in the two branches [1]. An important consequence of this is that the number of parameters to be estimated in a fractionally spaced channel model does not increase while the number of equations do, due to the extra data points. The estimates of the channel filters $H_1(q^{-1})$ and $H_2(q^{-1})$ can thus potentially be improved with fractionally spaced sampling.

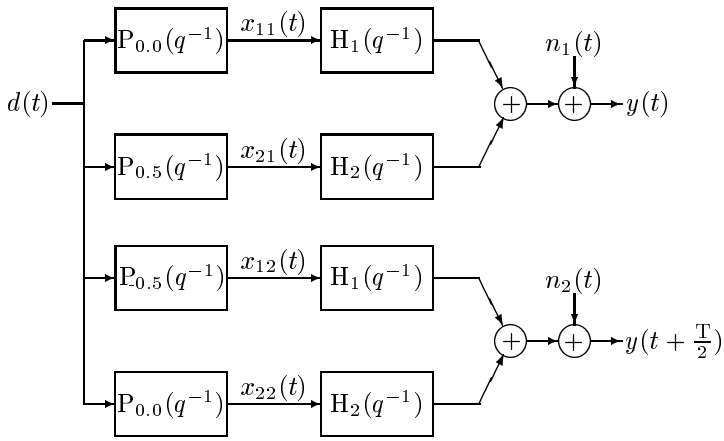

Figure 4: *Discrete channel model with fractionally spaced sampling and multiple pulse shaping filters per sampling branch.*

When moving to multiple users, as shown in Figure 5, the number of unknowns increases linearly with the number of users. The number of equations however remains constant. Here an economical parametrization of the channel becomes increasingly important.

3. CHANNEL ESTIMATION

We exemplify the channel estimation with two users, two branches per modeling branch and an oversampling factor of two. The equations can however easily be extended to more users, any amount of oversampling and any number of pulse shaping filters per branch.

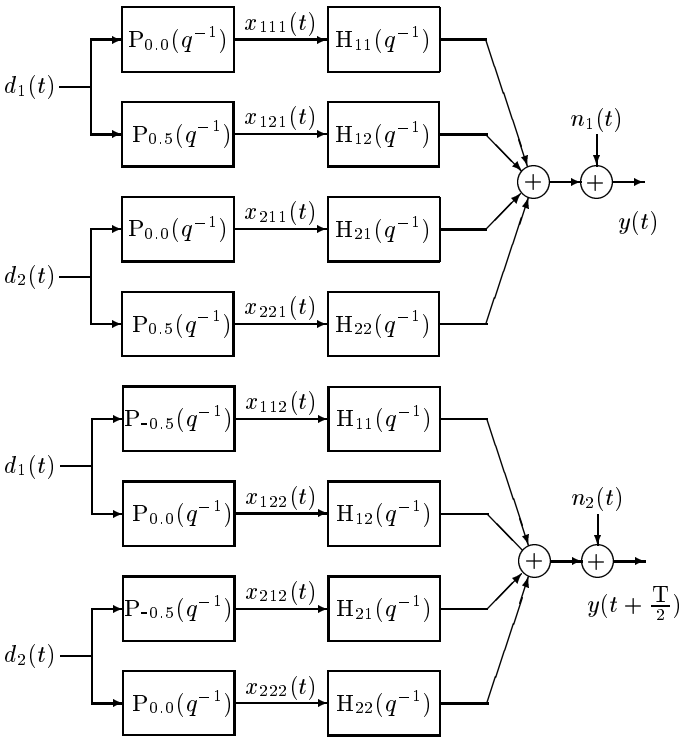

Figure 5: *Example multi-user channel model for two users transmitting the messages $d_1(t)$ and $d_2(t)$.*

The received signal $Y(t) = [y(t) \;\; y(t+\frac{\mathrm{T}}{2})]$ can be written as

$$Y(t) = \mathrm{H}(q^{-1})\mathrm{X}(t) \tag{1}$$

where

$$\mathrm{H}(q^{-1}) = [\mathrm{H}_{11}(q^{-1}) \;\; \mathrm{H}_{12}(q^{-1}) \;\; \mathrm{H}_{21}(q^{-1}) \;\; \mathrm{H}_{22}(q^{-1})] \tag{2}$$

and

$$\mathrm{X}(t) = \begin{bmatrix} x_{111}(t) & x_{112}(t) \\ x_{121}(t) & x_{122}(t) \\ x_{211}(t) & x_{212}(t) \\ x_{221}(t) & x_{222}(t) \end{bmatrix} = \begin{bmatrix} \mathrm{P}(q^{-1})d_1(t) \\ \mathrm{P}(q^{-1})d_2(t) \end{bmatrix} \tag{3}$$

with the pulse shaping matrix

$$\mathrm{P}(q^{-1}) = \begin{bmatrix} \mathrm{P}_{0.0}(q^{-1}) & \mathrm{P}_{-0.5}(q^{-1}) \\ \mathrm{P}_{0.5}(q^{-1}) & \mathrm{P}_{0.0}(q^{-1}) \end{bmatrix} \tag{4}$$

We see that we have a multiple-input multiple-output identification problem for estimating the channel $\mathrm{H}(q^{-1})$. This can be solved as a least squares problem. In order to form a system of equations we vectorize $H(q^{-1})$ and $X(t)$ in (1), giving

$$Y(t) = \mathrm{H}\mathcal{X}(t) \tag{5}$$

where

$$\mathrm{H} = [h_{11} \;\; h_{12} \;\; h_{21} \;\; h_{22}], \tag{6}$$

$$h_{ij} = [h_{ij0} \;\; h_{ij1} \;\; \ldots \;\; h_{ijnh}] \tag{7}$$

and

$$\mathcal{X}(t) = \begin{bmatrix} \bar{x}_{111}(t) & \bar{x}_{112}(t) \\ \bar{x}_{121}(t) & \bar{x}_{122}(t) \\ \bar{x}_{211}(t) & \bar{x}_{212}(t) \\ \bar{x}_{221}(t) & \bar{x}_{222}(t) \end{bmatrix}, \tag{8}$$

$$\bar{x}_{ijk}(t) = [x_{ijk}(t) \;\; \ldots \;\; x_{ijk}(t-nh)]^T \tag{9}$$

The unknown channel H can now be estimated as

$$\hat{\mathrm{H}} = \hat{R}_{Y\mathcal{X}}\hat{R}^{-1}_{\mathcal{X}\mathcal{X}} \tag{10}$$

where

$$\hat{R}_{Y\mathcal{X}} = \frac{1}{t_{max} - t_{min} + 1} \sum_{t=t_{min}}^{t=t_{max}} Y(t)\mathcal{X}^H(t), \tag{11}$$

$$\hat{R}_{\mathcal{X}\mathcal{X}} = \frac{1}{t_{max} - t_{min} + 1} \sum_{t=t_{min}}^{t=t_{max}} \mathcal{X}(t)\mathcal{X}^H(t) \tag{12}$$

The time instances t_{min} and t_{max} represents the first and last samples of the training data used.

4. EXAMPLE

For simplicity we have here chosen to illustrate the pulse shaping modeling with a raised cosine pulse with a roll-off factor 0.35, see Figure 6. In Figure 7, the channel estimation error for the T,T/2 and T/3 spaced interpolation can be seen.

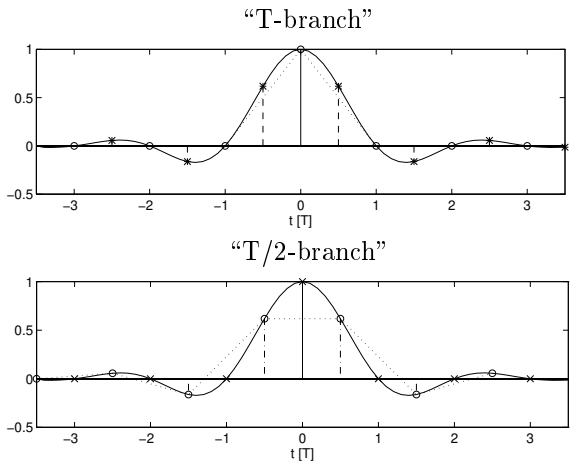

Figure 6: *Pulse shape, $p(t)$, and the taps of the pulse shaping filters, , $P_{0.0}(q^{-1})$ (o) and $P_{0.5}(q^{-1})$ (∗) for the "T-branch" and $P_{-0.5}(q^{-1})$ (o) and $P_{0.0}(q^{-1})$ (x) for the "T/2-branch".*

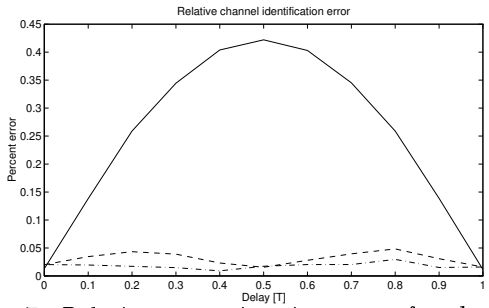

Figure 7: *Relative approximation error for the pulse for delays between 0 and T.*

Since it is advantageous to use multiple antennas in a multi-user scenario, we here use a receiver with 4 antennas. The number of users were ranged from one to 4. Each user had a channel with equal average power

Rayleigh fading taps at delays 0.00, 0.33, 0.67 and 1.00T. The taps for each user and each antenna element were independently fading. The channels were constant during each frame but independently fading between frames. Other parameters were: 18 training symbols, fractionally spaced sampling (two samples per symbol), T/2-spaced modeling. The SNR was 3dB and all users had equal average relative strength. After the channel to the users had been estimated a single user multi-channel MLSE [8] was applied to the received signal.

In Figure 8 the relative channel and BER for the multi-user channel estimation example can be seen for different number of users. First we can see that the joint LS channel estimation performs better than the single user LS channel estimation for two and three users. For four or more users the joint LS method is however worse. The method utilizing the pulse shaping information performs better than both the single and joint LS channel estimation. This is because this method has fewer parameters to estimate per user. For joint multi-user channel estimation, this becomes especially important as the number of users increases.

With pulse shaping, the channel spans about 4-6 symbol intervals depending on where we choose to truncate. In the LS methods we chose to use 4 taps and in the pulse shaping method we used 2 taps in each branch of $\mathrm{H}(q^{-1})$. The joint LS method will thus have 8 parameters *per user* to estimate with 36 equations (2×4 and 2×18 because of the fractionally spaced sampling). The pulse shaping method will have 4 parameters (2 per branch) *per user* to estimate with 36 equations. As the number of users increases this difference becomes more important. We can understand this by considering a limiting case when the number of users is so many that the joint LS method has more parameters than equations while this is not the case for the pulse shaping method. In this case the joint LS method will default while the pulse shaping method will still give some channel estimate. This is why, in Figure 8, we see that the joint LS method degrades in performance faster than the method utilizing the pulse shaping when the number of users increases.

5. SUMMARY

When estimating the channels to multiple users jointly with a short training sequence it is important to reduce the number of parameters per user. This is because the number of parameters increases linearly with the number of users while the number of equations remains constant, determined by the length of the training sequence.

We have demonstrated this by using a method utilizing the pulse shaping information in the transmitter and receiver in order to economize on the number of parameters to be estimated. In the multi-user channel estimation example shown, we can see that the method improves the channel estimate considerably, especially as the number of users increases.

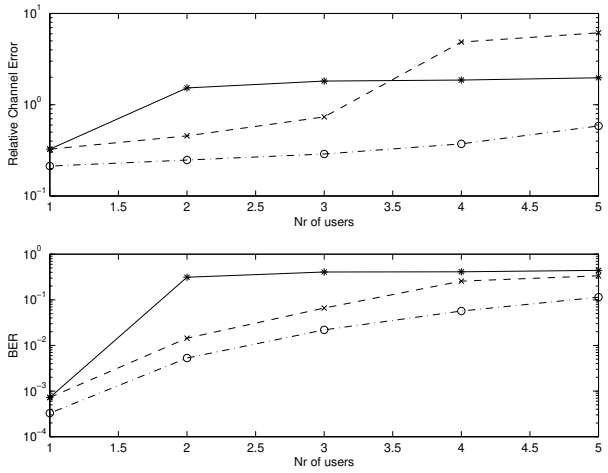

Figure 8: *Relative channel error and BER for the multi-user channel estimation example. Single user LS estimation (solid *), Joint muli-user LS (dashed x), Joint multi-user estimation utilizing pulse shaping information (dash-dotted o).*

6.REFERENCES

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