Bootstrap equalization

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ABSTRACT

We introduce bootstrapping as a generic method for improving channel equalization in TDMA cellular systems where the channels remain approximately constant over a transmitted burst. Bootstrapping implies that the detection is repeated, using an equalizer tuned with detected symbols. The procedure can be iterated to achieve even better performance. The initial equalizer is assumed to be tuned using a short known training sequence. When applied to experimental data from a DCS-1800 antenna array testbed, bootstrap equalization yields a performance improvement of 14 dB for a space-time maximum likelihood sequence detector and 17 dB for a space-time decision feedback equalizer in an interference limited scenario.

I. INTRODUCTION

In wireless cellular telephone systems, the channels often suffer from severe intersymbol and co-channel interference. In such systems, coherent detection and equalizers are necessary to achieve satisfactory performance. To design an equalizer, accurate estimation of the multipath channel and the noise spectrum is vital. A training sequence, known at the receiver, is typically used to estimate these unknown parameters.

The accuracy of the estimates is strongly dependent on the length of the training sequence. In systems currently in operation, the length of the training sequence has been chosen to give sufficiently accurate channel estimates for the deployed detectors, typically employing temporal filtering. Novel detectors, performing space-time filtering on the output of an antenna array can however gain from increased accuracy of the channel and noise spectrum estimates. This is due to the fact that these detectors have more degrees of freedom in their filtering. Hence, the performance of these algorithms can be considerably improved by using a longer training sequence [1, 2].

However, a longer training sequence leads to lower spectral efficiency, since the portion of the transmitted sequence which can be used to communicate unknown data is reduced. Also, new detectors may be desirable in present standards, where the length of training sequence cannot be altered.

One possibility to increase the accuracy of the channel estimates would be to use *decision-directed learning* [3] to update the channel recursively, using consecutive decisions from the detector. However, this would not improve the quality of the initial decisions.

Instead, we propose to design an equalizer based on the training sequence. We would then use this equalizer to detect all the transmitted symbols. These detected symbols can then be used to repeat the channel estimation and redesign the equalizer based on the estimates obtained in this second step. Hopefully, the tuning of the equalizer based on all the estimated symbols will be more accurate than the tuning based only on the training sequence, resulting in lower bit error rate in the second stage than in the first. If desired, this procedure can be repeated to further improve the performance. Such iterative *bootstrapping* was applied in [4], where it was presented as an integral part of a specific novel detector. However, bootstrapping leads to substantial performance improvements for other detectors as well. In this paper, we will investigate bootstrap equalization in conjunction with space-time versions of

- Maximum Likelihood Sequence Estimation (MLSE) implemented by means of the Viterbi algorithm.
- The Decision Feedback Equalizer (DFE).

Both detectors have been applied to experimental data collected at an antenna array testbed, which complies with the air interface standard of DCS-1800.

II. CHANNEL MODEL

Throughout the paper, we will consider discrete time channel models and detectors. A discrete time filter will be represented as a polynomial in the unit delay operator q^{-1}, as exemplified below:

$$\begin{aligned} v(k) &= A(q^{-1})u(k) \\ &= \left(A_0 + A_1 q^{-1} + \cdots + A_{na} q^{-na}\right) u(k) \\ &= A_0 u(k) + A_1 u(k-1) + \cdots + A_{na} u(k-na) \, . \end{aligned}$$

Filters will also be allowed to have terms with powers of q, the advance operator.

Multiple-input-single-output (MISO) filters will be represented as polynomial row vectors, whereas single-input-multiple-output (SIMO) filters will be represented as polynomial column vectors. Further, the complex conjugate transpose of a polynomial filter is defined as

$$(A(q^{-1}))^H \triangleq A^H(q) = A_0^H + A_1^H q + \cdots + A_{na}^H q^{na}$$

Note that this filter is *non-causal*.

A. A general single input-multiple output model

We consider a case where there is one transmitter and M receiver antennas. The transmitted symbol $d(k)$ propagates through the discrete-time channel $B_i(q^{-1})$ to receiver antenna i. The signal received at antenna i at the discrete time instant k is denoted $x_i(k)$ and can be expressed as

$$\begin{aligned} x_i(k) &= B_i(q^{-1})d(k) + v_i(k) \\ &= (B_i^0 + \cdots + B_i^{L_i} q^{-L_i})d(k) + v_i(k) \end{aligned}$$

The term $v_i(k)$ corresponds to noise and interference. The situation is depicted in Figure 1.

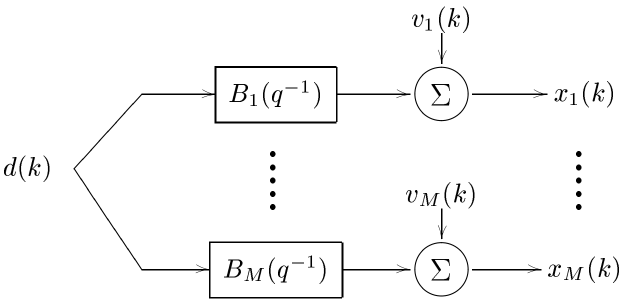

Figure 1: The general SIMO channel model. The signals $v_i(k)$ represent noise and co-channel interference.

To obtain a single-input-multiple-output (SIMO) model, we introduce the vectors

$$x(k) \triangleq \begin{pmatrix} x_1(k) & x_2(k) & \ldots & x_M(k) \end{pmatrix}^T \tag{1a}$$

$$v(k) \triangleq \begin{pmatrix} v_1(k) & v_2(k) & \ldots & v_M(k) \end{pmatrix}^T . \tag{1b}$$

The wide sense stationary vector of interference samples $v(k)$ can be both spatially and temporally colored and has the matrix-valued covariance function

$$\psi_{k-m} \triangleq E[v(k)v^H(m)] . \tag{2}$$

The vector of sampled antenna outputs $x(k)$ can now be expressed as

$$\begin{aligned} x(k) &= \begin{pmatrix} B_1(q^{-1}) \\ \vdots \\ B_M(q^{-1}) \end{pmatrix} d(k) + v(k) \\ &= \boldsymbol{B}(q^{-1})d(k) + v(k) \\ &= \mathbf{B}_0 d(k) + \cdots + \mathbf{B}_L d(k-L) + v(k) \end{aligned} \tag{3}$$

where we have defined the SIMO impulse response

$$\boldsymbol{B}(q^{-1}) \triangleq \begin{pmatrix} B_1(q^{-1}) \\ \vdots \\ B_M(q^{-1}) \end{pmatrix} \tag{4}$$

with individual matrix coefficients

$$\mathbf{B}_n \triangleq \begin{pmatrix} B_1^n \\ \vdots \\ B_M^n \end{pmatrix} . \tag{5}$$

In (3), $L \triangleq \max_i L_i$ represents the maximum delay spread over all subchannels.

III. DESCRIPTION OF THE DETECTORS

A. The spatio-temporal MLSE

The method of maximum likelihood sequence estimation searches for the transmitted symbol sequence $\{d(k)\}$ that maximizes the conditional probability of the received samples $\{x(k)\}$ given the transmitted signal $\{d(k)\}$. Let us assume that $v(k)$ can be modeled as temporally white but spatially colored Gaussian noise. Then the maximum likelihood estimate of the sequence is the sequence that maximizes

$$\begin{aligned} & P\left(\{x(k)\}_{k=1}^N \,|\{d(k)\}_{k=1}^N\right) \propto \\ & \exp\Big(-\sum_{k=1}^N [x(k) - \boldsymbol{B}(q^{-1})d(k)]^H \times \\ & \mathbf{R}_{vv}^{-1}[x(k) - \boldsymbol{B}(q^{-1})d(k)]\Big) , \end{aligned} \tag{6}$$

where $\boldsymbol{B}(q^{-1})$ is the channel defined in (4) and $\mathbf{R}_{vv} = \psi_0$ is the spatial covariance for the noise plus interference, defined in (2).

Maximizing (6) is equivalent to minimizing the log-likelihood function

$$\sum_{k=1}^N [x(k) - \boldsymbol{B}(q^{-1})d(k)]^H \mathbf{R}_{vv}^{-1}[x(k) - \boldsymbol{B}(q^{-1})d(k)] \tag{7}$$

Direct minimization of (7) is possible using the Viterbi algorithm. However, by rewriting (7), the complexity can be reduced without sacrificing performance.

Neglecting boundary effects, minimizing the log-likelihood metric (7) is equivalent [2] to maximizing the *matched-filter metric*

$$\begin{aligned} \mu_{MF} \triangleq\ & 2\mathrm{Re}\left\{\sum_{n=1}^N z^H(n)d(n)\right\} \\ & - \sum_{n=1}^N \sum_{m=1}^N d^*(n)\gamma_{n-m}d(m) , \end{aligned} \tag{8}$$

where the scalar signal $z(k)$ is defined by

$$z(k) \triangleq \boldsymbol{W}(q)x(k) . \tag{9}$$

The *multi-dimensional matched filter* (MMF) $\boldsymbol{W}(q)$ is defined by

$$\boldsymbol{W}(q) \triangleq \boldsymbol{B}^H(q)\mathbf{R}_{vv}^{-1} . \tag{10}$$

In (8), the coefficients γ_k are the coefficients of the complex conjugate symmetric double sided *metric* polynomial

$$\begin{aligned} \Gamma(q, q^{-1}) &\triangleq \boldsymbol{B}^H(q)\mathbf{R}_{vv}^{-1}\boldsymbol{B}(q^{-1}) \\ &= \gamma_{-L}q^L + \cdots + \gamma_0 + \cdots + \gamma_L q^{-L} \end{aligned} \tag{11}$$

where $\gamma_{-m} = \gamma_m^*$. The metric (8) can be recursively computed as

$$\begin{aligned} \mu_{MF}(k) =& \mu_{MF}(k-1) + \mathrm{Re}\Big\{d^*(k)\big[2z(k) \\ & -\gamma_0 d(k) - 2\sum_{m=1}^L \gamma_m d(k-m)\big]\Big\} \end{aligned} \tag{12}$$

and the maximization can thus be performed using the Viterbi algorithm. Note that the memory length in the matched filter metric (12) is the same as for the channel $\boldsymbol{B}(q^{-1})$. The Viterbi algorithm based on the matched filter metric will thus have the same number of states as the Viterbi algorithm based on the log-likelihood metric (7).

The spatio-temporal MLSE can now be divided into the MMF, $\boldsymbol{W}(q)$, followed by a scalar Viterbi algorithm using the metric defined by the polynomial, $\Gamma(q, q^{-1})$, see Figure 2. Note that the MMF $\boldsymbol{W}(q)$ is non-causal. In a real implementation, a sufficiently large delay has to be introduced.

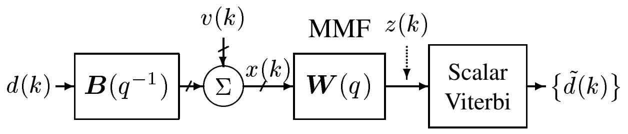

Figure 2: Spatio-temporal MLSE with multi-dimensional matched filter.

B. The spatio-temporal DFE

We shall use a spatio-temporal DFE with transversal feedforward and feedback filters as depicted in Figure 3.

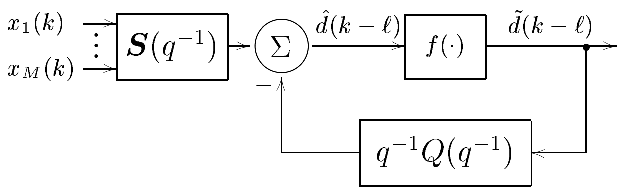

Figure 3: The structure of the space-time DFE.

The received signal samples $x(k)$ are filtered through the *feedforward filter* $\boldsymbol{S}(q^{-1}) = \mathbf{S}_0 + \mathbf{S}_1 q^{-1} + \cdots + \mathbf{S}_\ell q^{-\ell}$ and previously detected symbols $\tilde{d}(k-\ell-1)$ are filtered through the *feedback filter* $Q(q^{-1}) = Q_0 + Q_1 q^{-1} + \cdots + Q_{L-1} q^{-L+1}$ to form an estimate of the transmitted symbol $d(k-\ell)$. The feedforward filter has M inputs and a single output, whereas the feedback filter is a single-input-single-output filter. The DFE thus forms the decision variable

$$\hat{d}(k-\ell) = \boldsymbol{S}(q^{-1})x(k) - Q(q^{-1})\tilde{d}(k-\ell-1) \quad (13)$$

which is used as input to the decision device $f(\cdot)$ to produce a hard estimate $\tilde{d}(k-\ell)$ of the transmitted symbol $d(k-\ell)$.[1] The order of the feedforward filter equals the decision delay ℓ, whereas the order of the feedback filter should be one less than the extent of the intersymbol interference, $L-1$.

To make derivation of optimal equalizer coefficients feasible we shall adopt the usual assumption that all previous decisions affecting the current symbol estimate are *correct*, i.e. $\tilde{d}(k-\ell-n) = d(k-\ell-n),\ n = 1, \ldots, L$.

We will use a minimum mean square error decision feedback equalizer. The coefficients of such an equalizer are tuned to minimize

$$J = E[|d(k-\ell) - \hat{d}(k-\ell)|^2] \,. \quad (14)$$

To compute the coefficients of the MMSE optimal multi-input-single-output decision feedback equalizer, we will use estimates of the channel coefficients, the noise covariance function and the following theorem.

Theorem 1 *Consider the spatio-temporal DFE described by equation* (13)*, the channel model* (4) *and the noise description* (2)*. Assume the signal $d(k)$ is white with unit variance and uncorrelated with the noise $v(m)$ $\forall$ m. If all past decisions are correct, the matrix polynomials $\boldsymbol{S}(q^{-1})$ and $Q(q^{-1})$ of orders ℓ and $L-1$ respectively, minimizing* (14)*, are obtained as follows:*

1. *Solve the system of linear equations*

$$(\mathcal{B}\mathcal{B}^H + \Psi)\begin{pmatrix}\mathbf{S}_0^H \\ \vdots \\ \mathbf{S}_\ell^H\end{pmatrix} = \begin{pmatrix}\mathbf{B}_\ell \\ \vdots \\ \mathbf{B}_0\end{pmatrix} \quad (15)$$

where

$$\mathcal{B} = \begin{pmatrix}\mathbf{B}_0 & \ldots & \mathbf{B}_\ell \\ \vdots & \ddots & \vdots \\ 0 & \ldots & \mathbf{B}_0\end{pmatrix} \qquad \Psi = \begin{pmatrix}\psi_0 & \ldots & \psi_\ell \\ \vdots & \ddots & \vdots \\ \psi_{-\ell} & \ldots & \psi_0\end{pmatrix}$$

with respect to the matrix coefficients $\mathbf{S}_n^H$.

2. *Calculate the coefficients of the feedback filter according to*

$$Q_n = \sum_{i=0}^{\min(\ell, L-n-1)} \mathbf{S}_{\ell-i}\mathbf{B}_{i+n+1} \,. \quad (16)$$

Proof: See [5, 6]. ■

Remark. An optimally tuned DFE exploits the full spatio-temporal covariance function of the noise ψ_k $k = 0, \ldots, \ell$. However, the matrix valued interference spectrum cannot be reliably estimated with a short training sequence. Therefore, we are forced to use only the spatial color of the noise. [1]

IV. THE BOOTSTRAP IDEA

Let us assume that we have designed an equalizer using the training sequence. If we want to improve this initial equalizer, we could use decision-directed learning [3]. Since we are designing the detector based on a channel model, this would imply that we update the channel recursively, using consecutive decisions from the detector. As the decision from the detector are not necessarily correct, the estimated channel coefficients will not minimize the output MSE. In the spirit of [7], we would say that the estimated channel coefficients minimize the output *bootstrap mean-square error (BMSE)*:

[1] The soft estimate $\hat{d}(k-\ell)$ can be used to decode the channel code. A decision feedback equalizer thus provides soft information to the channel decoder without any additional complexity.

Definition 1 *The output bootstrap mean-square error is defined by*

$$J_{BMSE} \stackrel{\triangle}{=} E \parallel y(k) - \boldsymbol{B}(q^{-1})\tilde{d}(k) \parallel^2 \qquad (17)$$

where $\tilde{d}(k)$ is the output of the detector.

When decision directed learning is used to adapt the channel coefficients, (17) is minimized recursively using e.g. LMS or RLS. The current channel estimate is used to retune the detector, which outputs a new symbol decision, which is used to update the channel coefficients and so on. However, this would not improve the estimate of the first symbols in the burst, since these are detected using the initial equalizer.

Instead, we propose to minimize the output BMSE with iterative block processing. We would then use the initial equalizer to detect the *entire* symbol sequence and then minimize (17) with respect to the channel coefficients *while keeping the symbol decisions fixed*. If the initial error rate is sufficiently low these new estimates will provide better accuracy and lead to a detector with improved performance. If desired, the estimation and detection can be iterated.

V. EXPERIMENTS ON MEASURED DATA

To investigate the potential benefits of using the bootstrap algorithm outlined in Section IV, we applied the methods described in Section III to a set of uplink measurements.

A. The measurements

The measurements were performed on an antenna array testbed constructed by Ericsson Radio Systems AB and Ericsson Microwave Systems AB [8]. The testbed implemented the air interface of a DCS-1800 base station.

The array had eight antenna outputs. A conventional single sector antenna with two polarization diversity branches was also included in the measurement setup for two reasons: To facilitate comparison and to estimate the transmitted signal power.

The measurements were performed in downtown Düsseldorf, Germany.

In the measurements one mobile and one interferer were used. The transmit powers of the mobile and the interferer were adjusted so that the performance of the algorithms would be limited by the interference and not by the noise.

The performance of the algorithms will depend on the average *carrier-to-interference ratio* C/I. To estimate this quantity, we measured the received power at the sector antenna 1) when the mobile was active but the interferer was not and 2) when the interferer was active but the mobile was not. The ratio between these two measurement constitutes the C/I for this frame. This ratio was averaged over a segment of 100 frames. Due to the shadow fading, this average will vary between segments, and the performance of the algorithms can be addressed as a function of the average C/I.

The algorithms described in Section III were applied to the data recorded at the array antenna and at the single sector antenna when both mobile and interferer were transmitting.

B. Channel estimation

Both the spatio-temporal MLSE and the spatio-temporal DFE require the estimation of the multipath channel and the spatial covariance of the noise. We estimated the channel using the off-line least squares method.[2] The noise covariance matrix was computed from the residuals of the channel identification. The identified channel had five taps.

C. Results

C-1. The array antenna

We applied bootstrapped versions of the MLSE and the DFE to the experimental data from the array antenna. The results are shown in Figure 4.

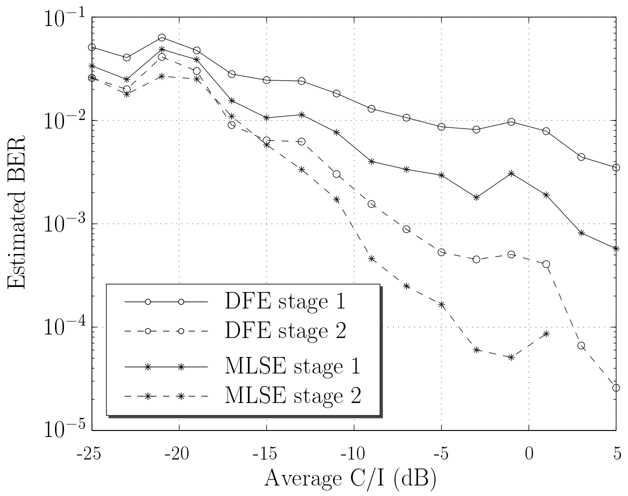

Figure 4: Results from the array antenna. Stage 1 refers to the equalizer designed using the training sequence, whereas stage 2 refers to the equalizer designed using the detections from stage 1.

From Figure 4, we see that it indeed pays to use the detected symbols to improve the estimation of channel and noise spectrum, especially for high C/I: at 5 dB, the second pass is 14 dB better than the first pass for the MLSE and 17 dB for the DFE!

C-2. The sector antenna

We also applied bootstrapped versions of the MLSE and the DFE to the experimental data from the sector antenna. The results are shown in Figure 5.

If we compare Figure 5 with Figure 4, we see that the gain for the sector antenna is much smaller. The antenna array introduces extra degrees of freedom, making powerful space-time processing possible. However, to make use of these extra degrees of freedom, more accurate estimation of the channel and noise spectrum is vital. For the two-branch diversity antenna, the performance of the space-time processing is not limited by the accuracy of the channel and noise spectrum estimates, and bootstrapping does not reduce the BER.

[2]Other channel estimation algorithms may yield better performance, see e.g. [9, 10]

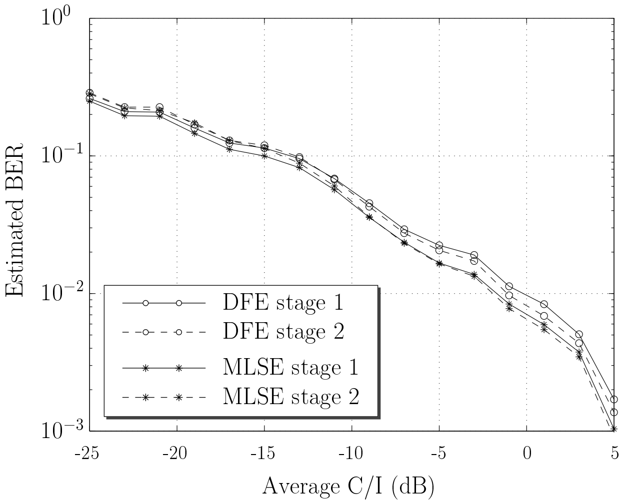

Figure 5: Results from the sector antenna. Stage 1 refers to the equalizer designed using the training sequence, whereas stage 2 refers to the equalizer designed using the detections from stage 1.

VI. DISCUSSION

In this paper, we have demonstrated the idea of bootstrap equalization for systems with time-invariant channel and noise statistics. We argue that the performance of detectors using antenna arrays is in fact limited by the accuracy of the estimates of channel and noise statistics, and that the performance of such detectors can be improved by using detected symbols to improve the required estimates. Subsequently, we use these new estimates to retune the detector and repeat the detection. Hopefully, the error rate from the second stage will be lower than the error rate resulting from the detector tuned using only the training sequence.

When introducing bootstrap equalization into a system it may also be possible to *reduce* the length of the training sequence. Bootstrapping can then be used to make up for the shorter training sequence, thereby increasing the spectral efficiency. Also, with access to an estimate of the entire transmitted sequence, it may be possible to reject strong interferers appearing outside the training sequence.

We have evaluated bootstrapped versions of MLSE and DFE on a set of experimental data. The results show that the performance is substantially improved when applied to measurements from an array antenna having eight outputs. However, when applied to a two-branch diversity sector antenna, the performance is only marginally improved.

VII. ACKNOWLEDGEMENTS

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REFERENCES

[1] Erik Lindskog, Anders Ahlén, and Mikael Sternad, “Spatio-temporal equalization for multipath environments in mobile radio applications,” in *Proceedings of the IEEE Vehicular Technology Conference*, vol. 1, Chicago, July 1995, pp. 399–403.

[2] Erik Lindskog, “Multi-channel maximum likelihood sequence estimation,” in *Proceedings of the 47th IEEE Vehicular Technology Conference*, vol. 2, Phoenix, Arizona, USA, May 5-7 1997, pp. 715–719.

[3] Robert W. Lucky, Jack Salz, and Edward J Weldon, Jr., *Principles of Data Communication*. New York: McGraw-Hill, 1968.

[4] Shilpa Talwar, Mats Viberg, and Arogyaswami Paulraj, “Blind separation of synchronous co-channel digital signals using an antenna array—part I: Algorithms,” *IEEE Transactions on Signal Processing*, vol. 44, no. 5, pp. 1184–1197, May 1996.

[5] Claes Tidestav, Mikael Sternad, and Anders Ahlén, “Reuse within a cell—multiuser detection or interference rejection?,” *IEEE Transactions on Communications*, submitted.

[6] Erik Lindskog. *Space-Time Processing for Equalization in Wireless Communication*, PhD thesis, Uppsala University, Uppsala, Sweden, in preparation.

[7] James Kennedy. *Equalization of digital communication channels using bootstrap mean-square error criterion*, PhD thesis, Stanford University, 1971.

[8] Sören Andersson, Ulf Forssén, Jonas Karlsson, Tom Witzschel, Peter Fischer, and Andreas Krug, “Ericsson/Mannesmann GSM field-trials with adaptive antennas,” in *Proceedings of the IEEE Vehicular Technology Conference*, vol. 3, Phoenix, USA, May 1997, pp. 1587–1591.

[9] Erik Lindskog, “Channel estimation exploiting pulse shaping information - a channel interpolation approach,”, Technical Report UPTEC 97138R, Signals and Systems Group, Uppsala University, PO Box 528, SE-751 20 Uppsala, Sweden, 1997, See http://www.signal.uu.se/Publications/preports.html.

[10] Boon Chong Ng and Mats Cedervall, “A structured channel estimator for maximum likelihood sequence detection in multipath fading channels,” *IEEE Communication Letters*, vol. 1, no. 2, pp. 52–55, March 97.