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Reduced Rank Space-Time Equalization

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ABSTRACT

In wireless communication systems with antenna arrays the spatio-temporal channel can often be described by a low-rank model. By exploiting this information, corresponding low rank equalizers with reduced complexity can be designed. By applying such low rank equalizers to a set of uplink measurements, we demonstrate that the performance loss associated with the lower complexity is small.

I. INTRODUCTION

The introduction of wireless communication receivers with multiple antenna elements makes it possible to perform space-time equalization. Such receivers efficiently combat intersymbol and co-channel interference caused by the spatio-temporal structure of the multipath channels. However, often the spatial and temporal dispersions in the channel is restricted to a low-dimensional subspace. We call such a channel a *reduced rank channel*.

One example of such a situation is when a partial response signal is sent through a physical channel with negligible delay spread. Due to the partial response modulation, the received signal will suffer from intersymbol interference. However, the combined spatio-temporal channel will lie in a one-dimensional subspace since all taps have the same spatial signature.

An equalizer for a channel with reduced rank will also have reduced rank in the sense that it will not exploit all spatio-temporal dimensions in the space-time filtering. This can be taken into account already in the design, leading to an equalizer with lower complexity. The space-time filtering in such an equalizer will consist of a (small) number of beamformers, each followed by a scalar temporal filter.

What we would call a rank one linear equalizer can be found in [1] and what in effect is a reduced rank decision feedback equalizer is presented in [2]. However, both these equalizers are tuned with iterative direct methods and do therefore not directly exploit the rank structure of the channel.

In this paper, we use reduced rank channel models to design rank reduced versions of the maximum-likelihood sequence estimator (MLSE) and the decision feedback equalizer (DFE). We also compare the performances of these low rank detectors to those of their full rank counterparts on a set of uplink measurements from an antenna array testbed.

II. REDUCED RANK CHANNEL APPROXIMATION

Throughout the paper, we will consider discrete time channel models and detectors. A discrete time filter will be represented as a polynomial in the unit delay operator q^{-1}, as exemplified below:

$$\begin{aligned} v(t) &= a(q^{-1})u(t) = \left(a_0 + a_1 q^{-1} + \cdots + a_{na} q^{-na}\right) u(t) \\ &= a_0 u(t) + a_1 u(t-1) + \cdots + a_{na} u(t-na) \ . \end{aligned}$$

Filters may also have terms with powers of the advance operator q.

Multiple-input-single-output (MISO) filters will be represented as polynomial row vectors, and single-input-multiple-output (SIMO) filters will be represented as polynomial column vectors. Further, the complex conjugate transpose of a filter (MISO or SIMO) is defined as

$$(\mathbf{a}(q^{-1}))^H \triangleq \mathbf{a}^H(q) = \mathbf{a}_0^H + \mathbf{a}_1^H q + \ldots + \mathbf{a}_{na}^H q^{na}$$

Note that this filter is *non-causal*.

A. Channel model

Consider the discrete-time, spatio-temporal FIR model of a wireless channel with L taps

$$\begin{aligned} \mathbf{x}(t) &= \mathbf{b}(q^{-1})d(t) + \mathbf{n}(t) \\ &= (\mathbf{b}_0 + \ldots + \mathbf{b}_{L-1} q^{-L+1})d(t) + \mathbf{n}(t) \end{aligned}$$

where $\mathbf{x}(t)$ is the received signal, $d(t)$ are the transmitted symbols and $\mathbf{n}(t)$ is noise and interference. The channel has one input and M outputs.

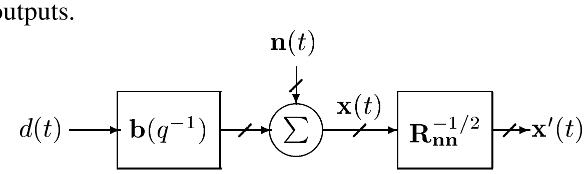

Figure 1: Channel with spatially colored noise and noise whitening filter.

Since the temporal spectrum of the noise cannot be reliably estimated [3], we will consider the noise to be temporally white but spatially colored with covariance matrix

$$\mathbf{R}_{\mathbf{nn}} = \mathrm{E}[\mathbf{n}(t)\mathbf{n}^H(t)] \ .$$

The relative strength of the signal compared to the noise and interference in the different spatial directions is of importance for the equalization. This information is contained in the *noise-whitened channel*. We therefore prewhiten the output of the channel $\mathbf{x}(t)$

with $\mathbf{R}_{\mathbf{nn}}^{-1/2}$ as in Figure 1, giving the equivalent channel model

$$\begin{aligned}\mathbf{x}'(t) &= \mathbf{b}'(q^{-1})d(t) + \mathbf{n}'(t) \\ &= (\mathbf{b}'_0 + \ldots + \mathbf{b}'_{L-1}q^{-L+1})d(t) + \mathbf{n}'(t)\end{aligned}$$

where

$$\mathbf{b}'(q^{-1}) = \mathbf{R}_{\mathbf{nn}}^{-1/2}\mathbf{b}(q^{-1}) \tag{1a}$$

$$\mathbf{x}'(t) = \mathbf{R}_{\mathbf{nn}}^{-1/2}\mathbf{x}(t) \tag{1b}$$

$$\mathbf{n}'(t) = \mathbf{R}_{\mathbf{nn}}^{-1/2}\mathbf{n}(t)\ . \tag{1c}$$

Working with this noise-whitened channel model will also be convenient in the following treatment.

B. Rank reduced model

We can express the channel $\mathbf{b}'(q^{-1})$ with the *channel matrix*

$$\mathbf{B}' = \begin{bmatrix} \mathbf{b}'_0 & \mathbf{b}'_1 & \ldots & \mathbf{b}'_{L-1} \end{bmatrix}\ . \tag{2}$$

By using a singular value decomposition, we can decompose the channel matrix as

$$\mathbf{B}' = \mathbf{U}\mathbf{V}^{\mathbf{H}} \tag{3}$$

where $\mathbf{U}$ consists of the left singular vectors $\mathbf{u}_k$ of $\mathbf{B}'$ and $\mathbf{V}$ consists of the right singular vectors $\mathbf{v}_k$ *with singular values included*:

$$\begin{aligned}\mathbf{U} &= \begin{bmatrix}\mathbf{u}_1 & \mathbf{u}_2 & \ldots & \mathbf{u}_K\end{bmatrix} \\ \mathbf{V} &= \begin{bmatrix}\mathbf{v}_1 & \mathbf{v}_2 & \ldots & \mathbf{v}_K\end{bmatrix}\end{aligned} \tag{4}$$

where $K = \min(M, L)$. The left singular vectors $\mathbf{u}_k$ are orthonormal, and the right singular vectors are orthogonal, sorted in descending order of magnitude, $\| \mathbf{v}_1 \| \geq \| \mathbf{v}_2 \| \geq \cdots \geq \| \mathbf{v}_K \|$.

We can use the singular value decomposition (3) and (4) to write the channel in polynomial form. For this purpose, we introduce the polynomials

$$v_k(q^{-1}) = v_{k0} + v_{k1}q^{-1} + \ldots + v_{k,L-1}q^{-L+1}$$

where v_{kj} is element j in $\mathbf{v}_k$. The noise-whitened channel can then be rewritten

$$\mathbf{b}'(q^{-1}) = \sum_{k=1}^{K} \mathbf{u}_k v_k(q^{-1})\ .$$

This decomposition is depicted in Figure 2.

With this model, the spatial properties of the channel are captured in the orthogonal spatial signatures $\mathbf{u}_k$, whereas the temporal dispersion is described by the polynomials $v_k(q^{-1})$.

We are now ready to define the rank of the channel.

Definition 1 *Consider the singular value decomposition* (3), (4) *of the channel matrix* (2). *If*

$$\| \mathbf{v}_k \| = 0\ \forall\ k > K_r$$

we say that the channel has rank K_r. If $K_r = K$ the channel is said to have full rank.

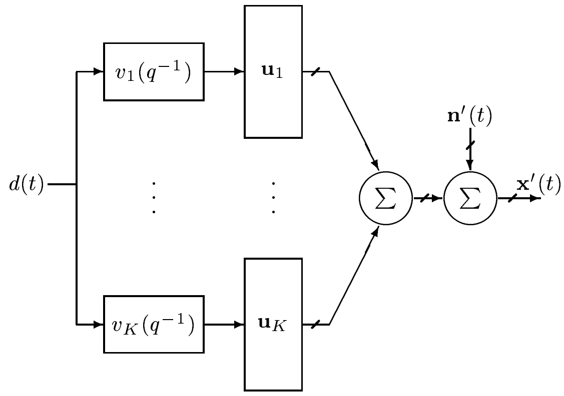

Figure 2: Decomposition of the noise-whitened channel.

The rank of a wireless channel is determined by the propagation environment. We believe that most practical wireless communication channels can be well approximated with a reduced rank model as

$$\mathbf{b}'(q^{-1}) \approx \sum_{k=1}^{K_r} \mathbf{u}_k v_k(q^{-1}), \quad K_r < K\ . \tag{5}$$

The corresponding rank reduced decomposition of the channel matrix will be

$$\mathbf{B}' \approx \mathbf{U}_r\mathbf{V}_r^H\ .$$

The rank reduced matrices $\mathbf{U}_r$ and $\mathbf{V}_r$ contain only the first K_r columns of the corresponding matrices $\mathbf{U}$ and $\mathbf{V}$. When the rank of the channel exceeds K_r, this ensures that we keep the principal components of the noise-whitened channel. Although this will not in general lead to the truly optimal rank reduced equalizer it is well motivated since we in this way keep the components of the channel with the highest signal to noise ratios.

III. REDUCED RANK SPACE-TIME MLSE

A. Spatio-temporal MLSE

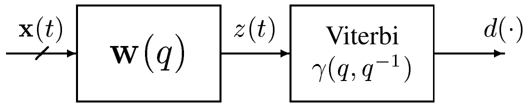

Figure 3: Space-time MLSE with multi-dimensional matched filter.

As shown in Figure 3, the space-time MLSE can be separated into a multidimensional matched filter (MMF), $\mathbf{w}(q)$, followed by a *scalar* Viterbi algorithm [4]. The MMF is given by

$$\mathbf{w}(q) = \mathbf{b}^H(q)\mathbf{R}_{\mathbf{nn}}^{-1} = \mathbf{b}'^H(q)\mathbf{R}_{\mathbf{nn}}^{-1/2}\ .$$

The signal after the MMF is processed by a scalar Viterbi algorithm in order to find the symbol sequence that maximizes the recursively defined matched filter metric

$$\begin{aligned}\mu_{MF}(t) = \mu_{MF}(t-1) + \mathrm{Re}\Big\{ & d^*(t)(2z(t) - \gamma_0 d(t) \\ & - 2\sum_{m=1}^{n\gamma} \gamma_m d(t-m))\Big\}\ .\end{aligned} \tag{6}$$

In (6),

$$z(t) = \mathbf{w}(q)\mathbf{x}(t)$$

is the output of the MMF, and γ_k are the coefficients of the double sided complex conjugate symmetric metric polynomial

$$\begin{aligned}\gamma(q, q^{-1}) &= \gamma^*_{n\gamma} q^{n\gamma} + \ldots + \gamma_0 + \ldots + \gamma_{n\gamma} q^{-n\gamma} \\ &= \mathbf{b}^H(q)\mathbf{R}_{\mathbf{nn}}^{-1}\mathbf{b}(q^{-1}) = \mathbf{b}'^H(q)\mathbf{b}'(q^{-1}) .\end{aligned}$$

The full rank MLSE is depicted in Figure 4.

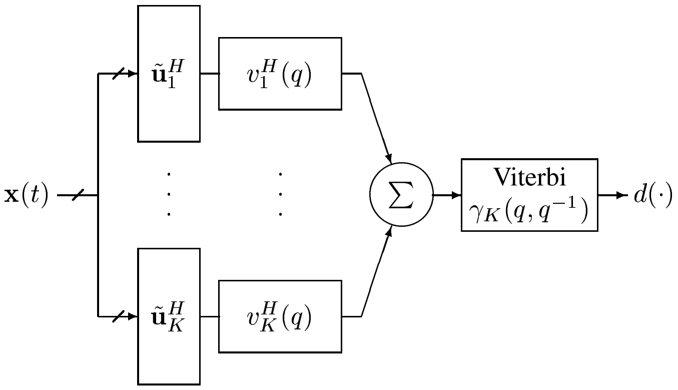

Figure 4: Full rank MLSE.

B. Rank reduction

When the channel has rank K_r, the beamformers $\tilde{\mathbf{u}}_{K_r+1}, \ldots, \tilde{\mathbf{u}}_K$ do not contribute to the input to the Viterbi algorithm. Therefore, we can rewrite the output of the MMF, using the rank reduced channel model (5):

$$\begin{aligned}z(t) &= \mathbf{w}(q)\mathbf{x}(t) = \mathbf{b}'^H(q)\mathbf{R}_{\mathbf{nn}}^{-1/2}\mathbf{x}(t) \\ &= \sum_{k=1}^{K_r} v_k^H(q)\mathbf{u}_k^H\mathbf{R}_{\mathbf{nn}}^{-1/2}\mathbf{x}(t) = \sum_{k=1}^{K_r} v_k^H(q)\tilde{\mathbf{u}}_k^H\mathbf{x}(t)\end{aligned}$$

where we have included the whitening of the received signal (1b) in the beamformers and defined

$$\tilde{\mathbf{u}}_k = \mathbf{R}_{\mathbf{nn}}^{-1/2}\mathbf{u}_k . \tag{7}$$

The MMF is followed by a scalar Viterbi algorithm using the metric defined by (6). However, when the channel is adequately described by the low rank model (5), the metric polynomial can be expressed as

$$\gamma_{K_r}(q, q^{-1}) = \sum_{k=1}^{K_r} v_k^H(q)v_k(q^{-1}) .$$

We call the resulting MLSE, depicted in Figure 5, a *reduced rank MLSE*.

IV. REDUCED RANK SPACE-TIME DFE

A. Space-time DFE

We assume that the received signal $\mathbf{x}(t)$ has been noise-whitened to form $\mathbf{x}'(t)$ and we use a space-time DFE as depicted in Figure 6. The received noise-whitened signal samples $\mathbf{x}'(t)$ are filtered through the feedforward filter, $\mathbf{s}'(q^{-1}) = \mathbf{s}'_0 + \mathbf{s}'_1 q^{-1} + \ldots + \mathbf{s}'_m q^{-m}$, and previously decided symbols are filtered through the feedback filter $Q(q^{-1}) = Q_0 + Q_1 q^{-1} + \ldots + Q_{nq} q^{-nq}$, in order to form an estimate $\hat{d}(t-m)$ of the symbol $d(t-m)$. The parameter m is the decision delay. The length of the feedback filter should be one less than the length of the channel.

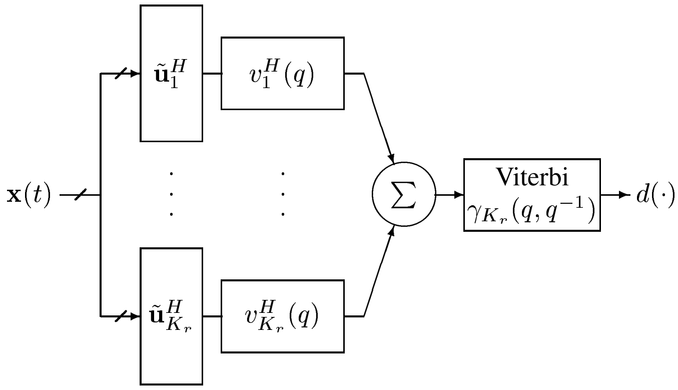

Figure 5: Reduced rank MLSE.

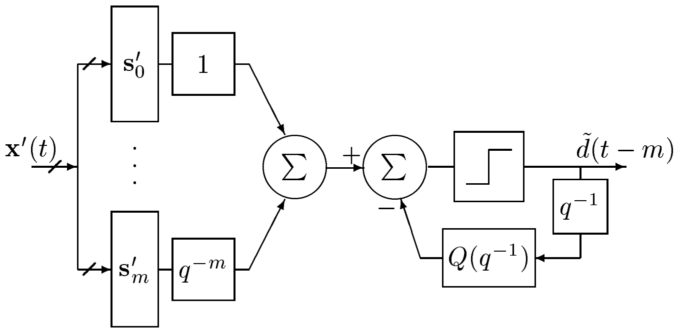

Figure 6: Structure of the general MISO FIR decision feedback equalizer.

When tuning the equalizer we assume that all previously decided symbols fed into the feedback filter are correct. We then tune the equalizer to minimize the mean square error

$$J = \mathrm{E}[|\hat{d}(t-m) - d(t-m)|^2] . \tag{8}$$

The feedforward filter coefficients minimizing (8) can then be computed [5, 6] by solving

$$(\mathcal{B}'\mathcal{B}'^H + \mathrm{I})\mathbf{s}'^H = \begin{pmatrix} \mathbf{b}'_m \\ \vdots \\ \mathbf{b}'_0 \end{pmatrix} \tag{9}$$

where

$$\mathbf{s}'^H = \begin{pmatrix} \mathbf{s}'_0 & \ldots & \mathbf{s}'_m \end{pmatrix}^H$$

and

$$\mathcal{B}' = \begin{pmatrix} \mathbf{b}'_0 & \mathbf{b}'_1 & \ldots & \mathbf{b}'_m \\ 0 & \mathbf{b}'_0 & \ldots & \mathbf{b}'_{m-1} \\ \vdots & \ddots & \ddots & \vdots \\ 0 & \ldots & 0 & \mathbf{b}'_0 \end{pmatrix} .$$

The feedback filter coefficients can be computed as

$$Q_k = \sum_{j=0}^{\min(m,L-1-k)} \mathbf{s}'_{m-j}\mathbf{b}'_{j+k+1} \; . \tag{10}$$

B. Rank reduction

Using the rank reduced channel model (5), a rank reduced DFE can be computed. We define the matrices

$$\mathcal{U}_r = \begin{pmatrix} \mathbf{U}_r & & 0 \\ & \ddots & \\ 0 & & \mathbf{U}_r \end{pmatrix}$$

$$\mathcal{V}_r = \begin{pmatrix} \mathbf{v}_{\text{row } 1} & & 0 \\ \vdots & \ddots & \\ \mathbf{v}_{\text{row } m} & \cdots & \mathbf{v}_{\text{row } 1} \end{pmatrix}$$

$$\mathcal{V}_{r,m} = \begin{pmatrix} \mathbf{v}_{\text{row } m} & \cdots & \mathbf{v}_{\text{row } 1} \end{pmatrix}$$

where

$$\mathbf{v}_{\text{row } k} = \begin{cases} \text{row } k \text{ in } \mathbf{V}_r & \text{if } k \le L \\ 0 & \text{if } k > L \end{cases} \; .$$

Equation (9) can now be written

$$(\mathcal{U}_r \mathcal{V}_r^H \mathcal{V}_r \mathcal{U}_r^H + \mathrm{I})\mathbf{s}'^H = \mathcal{U}_r \mathcal{V}_{r,m}^H \; .$$

We now change to the basis consisting of the columns of $[\mathcal{U}_r \;\; \mathcal{U}_{r,\perp}]$, where $\mathcal{U}_{r,\perp}$ is a matrix whose columns span the orthogonal complement to the span of $\mathcal{U}_r$. Noting that $\mathcal{U}_r^H \mathcal{U}_r = \mathrm{I}$ and $\mathcal{U}_{r,\perp}^H \mathcal{U}_r = 0$ we get the two equations

$$(\mathcal{V}_r^H \mathcal{V}_r + I)\mathcal{U}_r^H \mathbf{s}'^H = \mathcal{V}_{r,m}^H \tag{12a}$$

$$\mathcal{U}_{r,\perp}^H \mathbf{s}'^H = 0 \; . \tag{12b}$$

To solve (12a) we can consider the new unknown variable

$$\mathbf{g}^H = \begin{pmatrix} \mathbf{g}_0 & \ldots & \mathbf{g}_m \end{pmatrix}^H = \mathcal{U}_r^H \mathbf{s}'^H$$

and the corresponding equation

$$(\mathcal{V}_r^H \mathcal{V}_r + I)\mathbf{g}^H = \mathcal{V}_m^H \; .$$

If $\mathbf{g}$ satisfies this equation we see that

$$\mathbf{s}'^H = \mathcal{U}_r \mathbf{g}^H \tag{13}$$

solves both (12a) and (12b).

We can now reformulate (13) to get an expression for $\mathbf{s}'(q^{-1})$. Due to the special structure of $\mathcal{U}_r$, we obtain

$$\mathbf{s}'(q^{-1}) = \sum_{k=1}^{K_r} g_k(q^{-1})\mathbf{u}_k^H$$

where we have defined

$$g_k(q^{-1}) = g_{0k} + g_{1k}q^{-1} + \cdots + g_{mk}q^{-m}$$

and where g_{kj} is element j in $\mathbf{g}_k$.

Finally, we include the noise whitening filter from (1b) in the feedforward filter of the DFE:

$$\mathbf{s}(q^{-1}) = \mathbf{s}'(q^{-1})\mathbf{R}_{\mathbf{nn}}^{-1/2} = \sum_{k=1}^{K_r} g_k(q^{-1})\tilde{\mathbf{u}}_k^H$$

where $\tilde{\mathbf{u}}_k$ is defined in (7). The feedback filter can then be computed from the feedforward filter by using (10). The resulting reduced rank DFE is depicted in Figure 7.

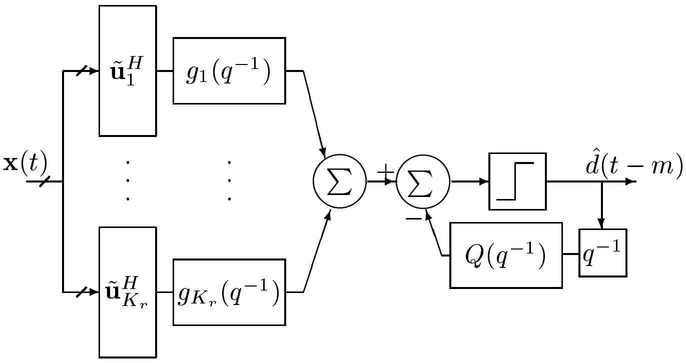

Figure 7: Reduced rank DFE.

V. COMPLEXITY SAVINGS

A. Execution complexity

The execution of the reduced rank filters will typically require less computations than the execution of the corresponding full rank filters. The complexity of the execution of the full rank (FR) and the reduced rank (RR) multidimensional matched filter (MMF) in the MLSE is

$$C_{\text{FR, MMF execution}} = N_d M L \; cu$$
$$C_{\text{RR, MMF execution}} = N_d K_r (M + L) \; cu \; .$$

Here N_d is the number of symbols equalized, M is the number of antenna elements and L is the channel length. The *complexity unit, cu*, is the complexity of one complex multiplication and one complex addition.

The corresponding complexities for the execution of the feedforward filter in the DFE is

$$C_{\text{FR, S execution}} = N_d M (m+1) \; cu$$
$$C_{\text{RR, S execution}} = N_d K_r (M + m + 1) \; cu$$

where m is the decision delay in the feedforward filter in the DFE.

B. Tuning complexity

Reducing the rank of the MMF for the MLSE will require some extra computations. This is however negligible in comparison with the complexity savings in the MLSE execution.

The tuning of the reduced rank DFE will in general be simpler than the tuning of the full rank DFE, since the system of linear equations (12a) has fewer unknowns than the corresponding system of linear equations (9). The complexity of solving for the feedforward coefficients is

$$C_{\text{FR, S solving}} \approx (M(m+1))^3/6\ cu$$
$$C_{\text{RR, S solving}} \approx (K_r(m+1))^3/6\ cu$$

for the full and reduced rank DFE:s, respectively.

VI. EXPERIMENTS ON MEASURED DATA

We have applied full and rank reduced versions of the spatio-temporal MLSE and the DFE to a set of uplink measurements.

A. The measurements

The measurements were performed on an antenna array testbed designed by Ericsson Radio Systems AB and Ericsson Microwave Systems AB [7]. The testbed implemented the air interface of DCS-1800. The array had eight antenna outputs. The measurements were performed in downtown Düsseldorf, Germany.

In the measurements one mobile and one interferer were used, and their transmit powers were adjusted so that the performance of the algorithms would be limited by interference and not by noise.

B. Algorithms

Both the MLSE and the DFE require the estimation of the multipath channel and the spatial covariance of the noise. We estimated a full rank, five tap model of the channel using the off-line least squares method. Reduced rank models of the channel were obtained by making low rank approximations of the full rank estimate as in (5).[1] The noise covariance matrix was computed from the residuals of the channel identification.

C. Results

We applied rank 1 and rank 2 versions of the MLSE and the DFE to the experimental data from the array antenna, and compared their performances to those of their full rank (rank 5) counterparts. The results are shown in Figure 8.

For the MLSE we see that the rank one version performs almost as good as the full rank MLSE. The rank two version has a BER *very* close to the full rank MLSE.

For the DFE the rank one version has some loss in performance. The rank two version conforms better with the performance of the full rank DFE, showing only a small loss in performance.

VII. CONCLUSIONS

By applying the rank reduction, the complexity in the execution of spatio-temporal equalizers can be reduced. In the case of the spatio-temporal DFE, the tuning of the equalizer parameters can also be simplified. The experimental study presented in this paper demonstrates that for practical wireless communication channels, reduced rank equalizers may provide adequate performance.

[1]The low rank property may also be explicitly included in the channel estimation by applying the method described in [8].

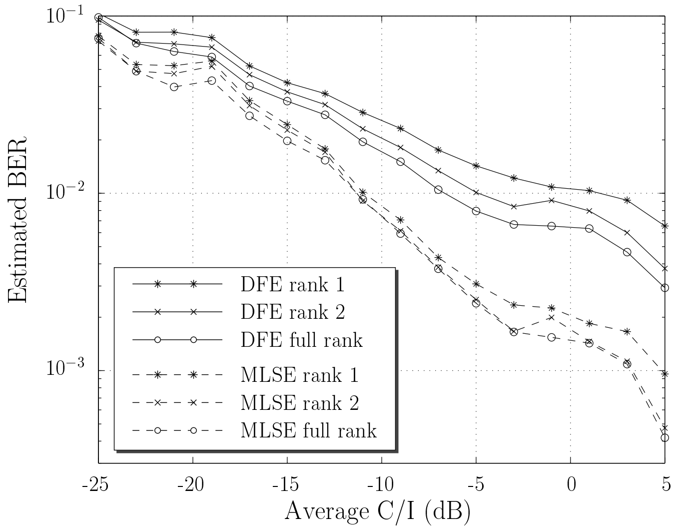

Figure 8: Performance of the full and the reduced rank equalizers.

VIII. ACKNOWLEDGEMENTS

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