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MULTI-CHANNEL MAXIMUM LIKELIHOOD SEQUENCE ESTIMATION

Erik Lindskog

Signals and Systems Group, Uppsala University, Box 528, S-751 20 Uppsala, Sweden
Phone: +46-18-183074, Fax: +46-18-555096, Email: el@signal.uu.se

ABSTRACT

In mobile radio communications, antenna arrays can be used to improve the quality and/or the capacity of the communication system. The combination of an antenna array and maximum likelihood sequence detection (MLSE) is studied here. Different realizations of the multi-channel MLSE are presented. Although equivalent in performance, it is pointed out that one of them, the multi-dimensional matched filter approach, is superior in terms of computational complexity when more than one antenna is used. For completeness, temporally colored noise is included in the formalism.

I. INTRODUCTION

Antenna arrays can be used to improve the quality and/or the capacity of a mobile radio communication system. The array processing for the antenna array can be combined with channel equalization or symbol detection. Here we combine maximum likelihood sequence estimation (MLSE) [1] [2] with the processing for the antenna array. This can be realized with a multi-channel MLSE.

Several different, but basically equivalent, realizations of the multi-channel MLSE can be found in the literature [3] [4] [5] [6] [7]. Three approaches, the log-likelihood, noise whitening and multi-dimensional matched filtering approach, are presented here. Although these approaches are equivalent in terms of their performance, one of them, the multi-dimensional matched filter implementation, is superior when using more than one antenna, since it requires less computations.

Handling of spatially colored noise and interference is readily performed with the multi-channel MLSE. For a more complete treatment temporally colored noise and interference is also included.

II. DIFFERENT DERIVATIONS OF THE MULTI-channel MLSE

A. Notation and channel description

In the discussion below, a polynomial description of filters will be used. For example, a causal FIR-filter will be represented with a polynomial in the delay operator, q^{-1}, as exemplified below

$$\begin{aligned} v(t) = A(q^{-1})u(t) &= (a_0 + a_1 q^{-1} + \ldots + a_{na} q^{-na})u(t) \\ &= a_0 u(t) + a_1 u(t-1) + \ldots + a_{na} u(t-na) \end{aligned} \tag{1}$$

Filters will also be allowed to have term with powers of q, the advance operator.

MISO and SIMO filters are represented as polynomial row and column vectors, respectively. MIMO filters are represented as polynomial matrices.

The complex conjugate transpose of a filter is written as

$$(A(q^{-1}))^H = A^H(q) = a_0^H + a_1^H q + \ldots + a_{na}^H q^{na} \tag{2}$$

Note that the filter is also time reversed.

The received signal at the M receiving antennas, $y(t) = [y_1(t) \quad y_2(t) \quad \ldots \quad y_M(t)]^T$ can now be written

$$y(t) = B(q^{-1})d(t) + n(t) \tag{3}$$

where $B(q^{-1}) = [B_1(q^{-1}) \quad B_2(q^{-1}) \quad \ldots \quad B_M(q^{-1})]^T$ represents the causal FIR channels to the antenna elements for the transmitted scalar symbol sequence $d(t)$. The noise plus interference is represented by the vector $n(t)$.

B. Log-likelihood metric and noise whitening approach

The straightforward method of deriving the MLSE is to maximize the probability for the received sequence, $y(t)$, of length N. If the noise and interference, for simplicity, is assumed Gaussian, then this probability can be expressed:

$$\begin{aligned} &P\left(y(t), t \in [1, 2, \ldots, N] | \{d(t)\}_{t=1}^{N}\right) \propto \\ &\quad exp\Bigg(- \sum_{t=1}^{N} \left[[R^{-1/2}(q^{-1}) \left\{ y(t) - B(q^{-1})d(t) \right\} \right]^H \\ &\quad \times \left[R^{-1/2}(q^{-1}) \left\{ y(t) - B(q^{-1})d(t) \right\} \right] \Bigg) \end{aligned} \tag{4}$$

where $R^{-\frac{1}{2}}(q^{-1})$ is the inverse of the causal part of the spectral factorization of the noise plus interferer spectrum

$$R(q, q^{-1}) = R^{1/2}(q^{-1})R^{H/2}(q) = \sum_{m=-\infty}^{\infty} E[n(t)n^H(t-m)]q^{-m} \tag{5}$$

The filter $R^{-1/2}(q^{-1})$ whitens the noise samples $n(t) = \{y(t) - B(q^{-1})d(t)\}$ and gives them unit variance.

Maximizing the probability in (4), is equivalent to minimizing the log-likelihood metric

$$\begin{aligned} \mu_{LL} = &\sum_{t=1}^{N} \left[R^{-1/2}(q^{-1}) \left\{ y(t) - B(q^{-1})d(t) \right\} \right]^H \\ &\times \left[R^{-1/2}(q^{-1}) \left\{ y(t) - B(q^{-1})d(t) \right\} \right] \end{aligned} \tag{6}$$

By forming the filtered signal,

$$y'(t) = R^{-\frac{1}{2}}(q^{-1})y(t) \tag{7}$$

and the new channel

$$B'(q^{-1}) = R^{-\frac{1}{2}}(q^{-1})B(q^{-1}) \tag{8}$$

the log-likelihood metric can be expressed as

$$\mu_{LL} = \sum_{t=1}^{N} \left[y'(t) - B'(q^{-1})d(t)\right]^H \left[y'(t) - B'(q^{-1})d(t)\right] \tag{9}$$

The metric can be recursively computed according to

$$\mu_{LL}(t) = \mu_{LL}(t-1) + \left[y'(t) - B'(q^{-1})d(t)\right]^H \times \left[y'(t) - B'(q^{-1})d(t)\right] \tag{10}$$

This metric can be minimized by using the Viterbi algorithm [8], replacing the standard scalar metric computation with the above vector formulation. Approaches similar to this can for example be found in [6], [5] and [7]. Since the Viterbi algorithm here works with a vector input we call it a *vector Viterbi*. The block diagram for an MLSE using this approach is depicted in Figure 1a.

If desired, we can move the whitening filter outside the vector Viterbi algorithm as shown in Figure 1b. We call this the noise whitening approach.

C. Multi-dimensional matched filter approach

Another formulation of the multi-channel MLSE is to formulate it in terms of a multi-dimensional matched filter as in [3] and [4]. These formulations can be seen as generalizations of [1] to the case with multiple channels. Using $y'(t)$ and $B'(q^{-1})$ (with coefficients b'_k) from equations (7) and (8), the matched filter version can be derived from the log-likelihood metric in (6) as:

$$\begin{aligned}
\mu_{LL} &= \sum_{t=1}^{N} \left[R^{-1/2}(q^{-1})\left\{y(t) - B(q^{-1})d(t)\right\}\right]^H \\
&\times \left[R^{-1/2}(q^{-1})\left\{y(t) - B(q^{-1})d(t)\right\}\right] \\
&= \sum_{t=1}^{N} \left[R^{-1/2}(q^{-1})y(t)\right]^H \left[R^{-1/2}(q^{-1})y(t)\right] \\
&-2\mathrm{Re}\left\{\sum_{t=1}^{N} \left[R^{-1/2}(q^{-1})y(t)\right]^H R^{-1/2}(q^{-1})B(q^{-1})d(t)\right\} \\
&+\sum_{t=1}^{N} \left[R^{-1/2}(q^{-1})B(q^{-1})d(t)\right]^H \left[R^{-1/2}(q^{-1})B(q^{-1})d(t)\right] \\
&= f(y(\cdot)) - 2\mathrm{Re}\left\{\sum_{t=1}^{N}\sum_{k} y'^H(t)b'_k d(t-k)\right\} \\
&+\sum_{t=1}^{N}\sum_{k}\sum_{l}[b'_k d(t-k)]^H[b'_l d(t-l)] \\
&= /\ n = t-k,\ m = t-l \text{ and } t-n \to t \text{ in term } 2/ = \\
&= f(y(\cdot)) - 2\mathrm{Re}\left\{\sum_{n=1}^{N}\sum_{t}[b_t'^H y'(n+t)]^H d(n)\right\} \\
&+\sum_{n=1}^{N}\sum_{m=1}^{N} d^*(n)(\sum_{t} b_{t-n}'^H b'_{t-m})d(m)] + e_{\text{border}} \\
&= -2\mathrm{Re}\left\{\sum_{n=1}^{N} z^H(n)d(n)\right\} - \sum_{n=1}^{N}\sum_{m=1}^{N} d^*(n)\gamma_{n-m}d(m) \\
&+f(y(\cdot)) + e_{\text{border}}
\end{aligned}$$

where the scalar signal $z(t)$ is defined by

$$z(t) = W(q, q^{-1})y(t) = B^H(q)R^{-1}(q, q^{-1})y(t) \tag{11}$$

and γ_k is the k:th coefficient of the metric polynomial

$$\Gamma(q, q^{-1}) = B^H(q)R^{-1}(q, q^{-1})B(q^{-1}) \tag{12}$$

with the coefficients numbered as

$$\Gamma(q, q^{-1}) = \gamma_{-n\gamma}q^{n\gamma} + \ldots + \gamma_0 + \ldots + \gamma_{n\gamma}q^{-n\gamma} \tag{13}$$

The term $f(y(\cdot))$ does not depend directly on the transmitted symbols, $d(\cdot)$, and can therefore be dropped. The term e_{border} is a correction term that only depends on the values of $y(t)$ and $d(t)$ for t close to 1 or N. Neglecting the "border effects" and changing the sign such that the matched filter metric is to be *maximized* gives

$$\mu_{MF} = 2\mathrm{Re}\left\{\sum_{n=1}^{N} z^H(n)d(n)\right\} - \sum_{n=1}^{N}\sum_{m=1}^{N} d^*(n)\gamma_{n-m}d(m) \tag{14}$$

When $\Gamma(q, q^{-1})$ is complex conjugate symmetric, this metric can be recursively computed as

$$\mu_{MF}(t) = \mu_{MF}(t-1) + \mathrm{Re}\Big\{d^*(t)(2z(t) - \gamma_0 d(t) -2\sum_{m=1}^{n\gamma+1}\gamma_m d(t-m))\Big\} \tag{15}$$

We call the filter, $W(q, q^{-1}) = B^H(q)R^{-1}(q, q^{-1})$, a multidimensional matched filter (MMF). Note also that the memory length in the Viterbi algorithm using the matched filter metric in (14), is the same as for the Viterbi algorithm using the log-likelihood metric in (9).

An advantage with this latter version of the multi-channel MLSE, is that due to the matched filtering, only a scalar Viterbi algorithm is required. This reduces the the complexity of the metric computation in the Viterbi algorithm.

A block diagram of the MLSE using the multi-dimensional matched filter approach can be seen in Figure 1c. Note also that the multi-dimensional matched filter can be broken up into a noise whitening part, $R^{-1/2}(q^{-1})$, and a filter matched to the overall resulting SIMO channel, $B^H(q)R^{-H/2}(q)$. This can be seen in Figure 1d.

In the case with temporally colored noise, a problem arises. Unless the noise is colored by an AR-filter, the filter $R^{-1}(q, q^{-1})$ will have a double sided infinite impulse response. This results in an infinite memory length in the Viterbi algorithm. Either only the metric or both the metric and the MMF has to be truncated. The same will be required for the log-likelihood and noise whitening approach.

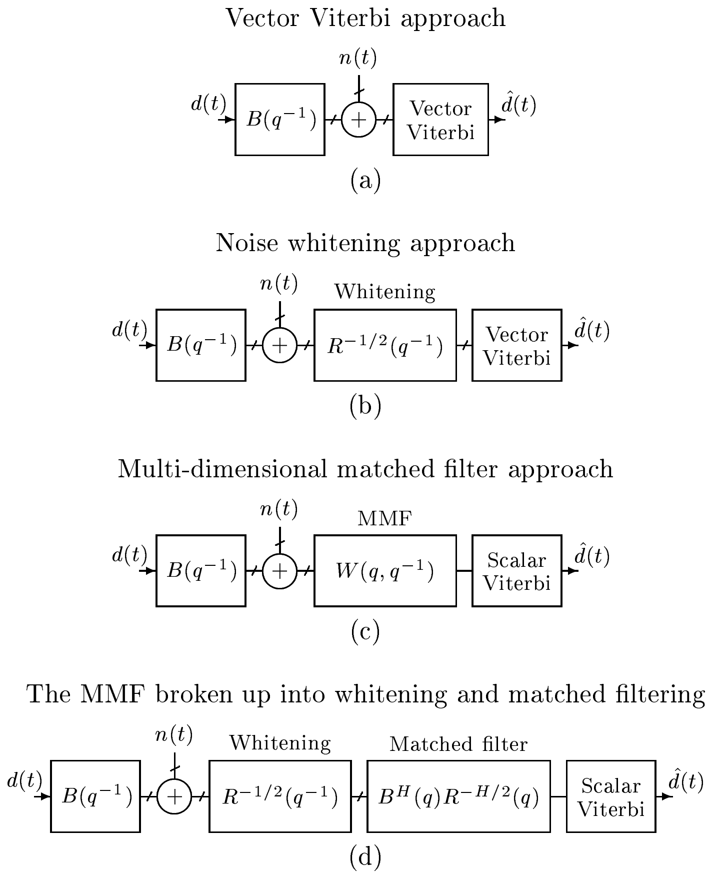

Figure 1: Different approaches for multi-channel MLSE.

III. COMPUTATIONAL COMPLEXITY

Let us assume that the channel $B'(q)$ is approximated by an FIR filter with $nb + nr + 1$ taps. If L is the number of symbols in the alphabet, a Viterbi algorithm then requires, $L^{nb+nr+1}$ metric updates for each detected symbol.

If the channel is assumed stationary, the computational complexity of the log-likelihood metric and the whitening filter approach, measured by the number of complex multiplications and additions per symbol detected, will be

$$C_{LL} \approx M^2(nr+1) + M(nb+2)L^{(nb+nr+1)} \tag{16}$$

where M is the number of channels.

Because the matched-filter metric operates a scalar Viterbi-algorithm, the complexity for this approach will be

$$C_{MF} \approx M(2nr+nb+1) + (nb+2)L^{(nb+nr+1)} \tag{17}$$

It can be seen from these expressions, that if the number of antennas is more than one, the matched-filter metric has a definite advantage over the log-likelihood metric.

IV. TUNING THE MULTI-DIMENSIONAL MATCHED FILTER

The multi-dimensional matched filter, $W(q, q^{-1})$, and the metric polynomial, $\Gamma(q, q^{-1})$, can be tuned in a few different ways.

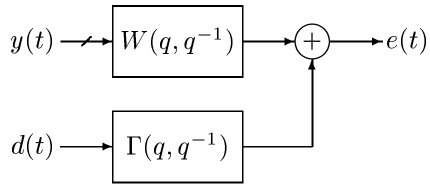

Figure 2: MMF filter tuning.

A. Direct tuning

In [9] and [3], generalizations of the direct approach in [1] are presented. The coefficients of a feedforward filter and the coefficients of an non-causal feedback filter are tuned to minimize the mean square error (MSE) of the error signal,

$$e(t) = W(q, q^{-1})y(t) - \Gamma(q, q^{-1})d(t) \tag{18}$$

See Figure 2. By this minimization, noise whitening and matched filtering will be performed by $W(q, q^{-1})$, while $\Gamma(q, q^{-1})$ will contain the overall impulse response. The polynomial row vector, $W(q, q^{-1}) = [W_1(q, q^{-1}) \ \ldots \ W_M(q, q^{-1})]$, is a MISO FIR filter and $\Gamma(q, q^{-1})$ is a double sided, complex conjugate symmetric, non-causal FIR-filter with the middle coefficient, γ_0, constrained to be equal to one. That is,

$$\Gamma(q, q^{-1}) = \gamma_{-n\gamma}q^{+n\gamma} + \ \ldots \ + 1 + \ \ldots \ + \gamma_{n\gamma}q^{-n\gamma} \tag{19}$$

with $\gamma_{-k} = \gamma_k^*$.

It is natural to chose the structure of the feedforward filter, $W(q, q^{-1})$, consistent with an ideal MMF with a truncated noise plus interference spectrum, $B^H(q)\tilde{R}^{-1}(q, q^{-1})$. The spectrum, $\tilde{R}^{-1}(q, q^{-1})$, here represents a truncated version of $R^{-1}(q, q^{-1})$. The filter $W(q, q^{-1})$ will thus be non-causal or anti-causal, since $B^H(q)$ is anti-causal and $\tilde{R}^{-1}(q, q^{-1})$ either a matrix constant or a double-sided polynomial matrix. It is also natural to choose the number of coefficients in $\Gamma(q, q^{-1})$ consistent with the structure chosen for $W(q, q^{-1})$, according to $\Gamma(q, q^{-1}) = B^H(q)\tilde{R}(q, q^{-1})B(q^{-1})$.

The estimates $\hat{W}(q, q^{-1})$ and $\hat{\Gamma}(q, q^{-1})$ can be found either adaptively or by solving a system of equations formed directly from the training data.

When the true filter orders are used and the training sequence is long enough, the MMF will be contained in the estimate $\hat{W}(q, q^{-1})$, up to a multiplicative constant, and the corresponding metric to be used in the Viterbi algorithm will be contained in the estimate $\hat{\Gamma}(q, q^{-1})$ [9].

A problem arises if the available training sequence is short. If for instance the number of training symbols is smaller than the number of coefficients in the filters, then the coefficients cannot be determined uniquely. A regularization of the equations can be introduced. By adding artificial noise into the system of equations, a solution can be computed, but it will in general be inferior to the true matched filter.

By adjusting the number of coefficients in $W(q, q^{-1})$ and $\Gamma(q, q^{-1})$, the ability to combat temporally colored interference can be varied. Adding more coefficients increases the filters temporal noise whitening as well as matched filtering capability at the expense of more degrees of freedom and a longer memory in the metric.

B. Indirect MMF tuning

In the indirect approach the channel, $B(q^{-1})$, is first identified. An estimate of the noise plus interference spectrum, $R(q, q^{-1})$, can then be formed using the residuals from the identification procedure.

The channel can be identified in different ways. We will here assume that a short training sequence is available.

The channel, $B(q^{-1})$, can for example be identified with a standard least squares method. It is also possible to parametrize each vector tap in $B(q^{-1})$ in terms of directions of arrival and complex amplitudes of a finite number of paths. This parametrization can then be used in order to attempt to improve the initial least squares channel estimate. Examples of both these methods can be found in [10].

In [11] a method is presented that makes use of the knowledge of the pulse shaping function in the modulation. This will further improve the channel estimates.

It is also possible, using the pulse shaping function, to parametrize the channel, $B(q^{-1})$, in terms of directions of arrival, complex amplitudes and delays of a finite number of paths. These parameters can then be estimated with a maximum likelihood method [12].

Theoretically, if the spectrum is invertible, estimates of the MMF, $W(q, q^{-1})$, and of the metric polynomial, $\Gamma(q, q^{-1})$, can then be formed as

$$\hat{W}(q, q^{-1}) = \hat{B}^H(q)\hat{R}^{-1}(q, q^{-1}) \tag{20}$$

$$\hat{\Gamma}(q, q^{-1}) = \hat{B}^H(q)\hat{R}^{-1}(q, q^{-1})\hat{B}(q^{-1}) \tag{21}$$

where the "hat" marks quantities derived from the estimated channel, $\hat{B}(q^{-1})$, or the estimated noise spectrum, $\hat{R}(q, q^{-1})$.

Two problems arises. First, the filter $\hat{R}^{-1}(q, q^{-1})$ will in general be a double sided IIR filter. In order to have a finite memory in the Viterbi algorithm, the metric has to be truncated. Second, unless the training sequence is long, $R(q, q^{-1})$ cannot be properly identified. A good choice that produces a solution to both problems is to identify and use only the spatial correlation of the noise, i.e. the zeroth matrix coefficient R_0.

C. Indirect MMSE tuning

An alternative indirect way of tuning the multi-dimensional matched filter is to perform the minimization of the MSE of the error signal in (18), but instead of forming the systems of equations directly from data, we form them from the identified channel, $\hat{B}(q^{-1})$, and the matrix coefficients of the estimated noise spectrum, $\hat{R}(q, q^{-1})$. The number of matrix coefficients of $\hat{R}(q, q^{-1})$ used, and the structure and length of the filters $W(q, q^{-1})$ and $\Gamma(q, q^{-1})$, affects the temporal noise whitening and matched filtering capabilities. By constraining the filter structures, the memory length in the Viterbi algorithm can be controlled.

If the same structure is used and only the spatial noise plus interference spectrum is utilized, the two indirect methods are equivalent. The methods differ only in the way they handle *temporally* colored noise plus interference.

The indirect MMSE tuning of the MMF has the potential advantage that it for a given filter structure, and a given noise plus interference spectrum used, finds filter coefficients that performs a compromise between noise whitening and matched filtering.

An interesting question to study, is if the indirect methods can handle a case with very low signal-to-interference ratio (SIR). It could be suggested, that very poor SIR would make identification of the channels to the individual antenna elements unfeasible. Although the quality of the identified channels may be compromised, the simulations for the scenario presented here do not show that the indirect methods suffer much from this.

V. SIMULATIONS

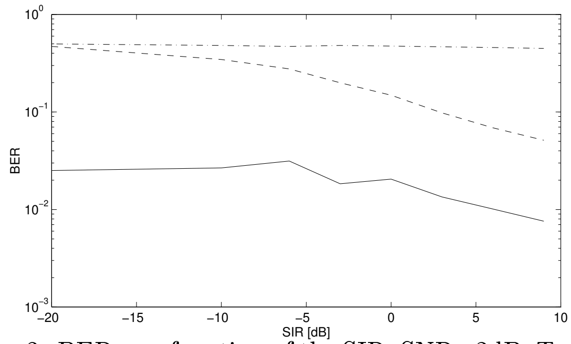

Figure 3: BER as a function of the SIR. SNR=2dB. Training sequence length=26. Indirect tuning using only spatial noise color (solid). Direct tuning with regularization (dashed) and without (dash-dotted).

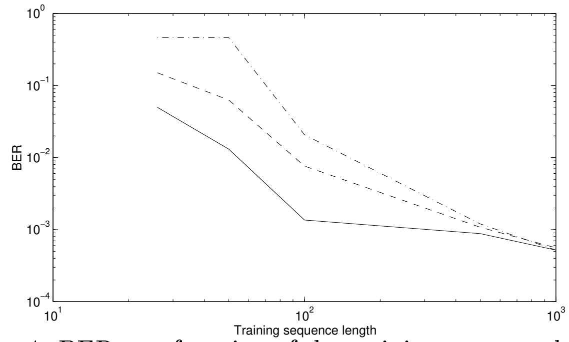

Figure 4: BER as a function of the training sequence length. SIR and SNR is 0 dB. Indirect tuning using only spatial noise color (solid). Direct tuning with regularization (dashed) and without (dash-dotted).

The purpose of the simulations presented here is to study how the indirect methods can handle poor signal to interference ratios and to demonstrate that the direct method has problems with short training sequences.

The algorithms compared are the indirect method (either one), when using only the spatial color of the noise plus interference, and two versions of the direct method. One without regularization and one with artificial noise with a variance equal to the real noise variance, added to the diagonal of the system of equations. In all cases 5 taps in $W(q,q^{-1})$ and 9 taps in $\Gamma(q,q^{-1})$, were used.

The channels for the indirect method were identified with the standard least squares method.

The algorithms were tested using a circular array with eight antennas equally spaced along a circle with a radius of 0.5 wave lengths. The desired signals arrives from the directions 0,30,-60 and 180 degrees. The respective channels are $1+0.5q^{-1}$, $0.5q^{-1}+0.8q^{-2}$, $0.5q^{-2}+0.2q^{-3}$ and $0.2q^{-3}+0.3q^{-4}$. Two-tap channels are chosen in order to simulate imperfect sampling timing or partial response modulation. Binary symbols, $d(t)=\pm 1$, are used. Co-channel interferers impinge on the antenna array, also through two tap channels, from the directions -30, 135 and -135 degrees. White Gaussian noise was also added.

In Figure 3, the BER for the different algorithms can be seen as a function of the SIR. It can be seen from the figure, that the indirect method does not suffer significantly in this scenario from the poor SIR conditions. The direct methods, of course, perform poorly here since the training sequence was only 26 symbols long.

In Figure 4, the BER is presented as a function of the length of the training sequence used. When the training sequence length increases the performance of the direct methods approaches that of the indirect method. The indirect method performs better since it focuses on the spatial color of the noise plus interference, which is the most important in this and many other antenna array scenarios.

VI. SUMMARY

The log-likelihood metric, the noise whitening and the matched filter approach, are all equivalent in terms of performance. The matched filter approach is however superior in terms of computational complexity when more than one antenna is used. The metric computation in the Viterbi part of the algorithm, is reduced by a factor equal to the number of antennas, when compared to the log-likelihood metric approach.

The indirect MMSE tuning presented, has interesting variability in its structure that allows tradeoffs between complexity and performance. The MMF:s ability to temporally whiten noise and match filter the signal, as well as controlling the memory length of the subsequent Viterbi-algorithm, can be varied. The usefulness of this for, for example a TDMA system with a short training sequence, remains to be assessed.

Although temporal whitening of noise plus interference is included in the formalism, the usefulness of this also remains to be determined. It is probably especially difficult to make use of the temporal color of the noise plus interference when only a short training sequence is available for the tuning.

If fractionally spaced sampling is required in order to get the sampling frequence up to, or above, twice the maximum relevant frequency content in the received signal, this can be accomplished by adding the fractionally spaced samples as signals from virtual sensors, adding extra channels.

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