ANTI-WINDUP COMPENSATORS FOR MULTIVARIABLE CONTROL SYSTEMS*

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Abstract

Model-based anti-windup compensation is here considered for multiple-input multiple-output (MIMO) systems. The aim will be to modify the dynamics of a control loop when actuators saturate, so that a good transient behaviour is attained after desaturation, while avoiding limit cycle oscillations and repeated saturations. It is of advantage if such effects can be controlled separately, while leaving the nominal dynamics unchanged when no actuators saturate. A controller structure with three degrees of freedom, with feedback from saturated control signals, is therefore proposed. The transfer function of this controller corresponds to that of a nominal controller, with two degree of freedom, as long as none of the actuators saturate.

The structure of the controller is selected such that the loop gain around the bank of saturations is made diagonal. The properties of the loop around each saturation can then be tuned separately. We propose one way of doing this, by means of solving a set of separate scalar $\mathcal{H}_2$ problems. The proposed approach is applicable to continuous-time as well as discrete-time systems. Although it is here presented for systems in input-output form, it can be used in state-space designs just as well.

1 Introduction

The problem of actuator saturation has led to a search for controllers which preserve the most important properties of linear closed-loop systems during and after the saturation events. This research has, over the years, resulted in a number of different anti-windup schemes and strategies [2]-[11]. Some of these schemes are also used for analysis and design of mode switching, e.g. when switching between different controllers. Problems associated with mode switching are often refered to as *bumpless transfer* (BT) problems.

Many of the proposed anti-windup strategies focus on adjusting the states of the controller during a saturation event. For reasons explained in [9], and more recently also in [3, 5, 6], there is, however, no guarantee that the *whole* system behaves acceptably during or after a saturation event, when only *controller*-state windup is prevented. Repeated saturations and limit cycles might occur.

To avoid such effects, the whole linear dynamics around the saturating elements, consisting of nominal controller elements, anti-windup filters and the plant, has to be taken into account. In the scalar case this can be accomplished in a Nyquist diagram: the loop gain around the saturating element could be adjusted so that it stays well away from the function $-1/Y(C)$, where $Y(C)$ is the describing function of the saturation nonlinearity [7, 9], see Figure 3. The aim of the present paper is to introduce a controller structure which makes it possible to generalize this technique for analysis and design to feedback systems with multiple measurements and control signals.

A key simplification is that the loop gain relevant for the saturation behaviour is made diagonal. The diagonal elements can then be adjusted in the same way as for a scalar system. Since our primary goal is to develop analytical tools which are easy to use also for multivariable systems, this property is highly desirable.

In section 2 we will present a controller structure with three degrees of freedom, useful for analysis and design of both anti-windup and bumpless transfer strategies. After that section, we will restrict attention to the anti-windup problem only.

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2 Representation of the system and the controller

We will consider a class of linear time-invariant (LTI) discrete-time[1] systems with p outputs, which are controlled by m inputs via memoryless nonlinearities. The linear systems are assumed controllable and observable, and their poles are assumed to be located in the closed unit disc $|z| \leq 1$. It can be shown, see [8], that such systems can always be stabilized by using constrained actuators. For our purpose, it will be convenient to parametrize the plant model in right rational fractional form with a diagonal denominator matrix, as

$$\begin{aligned} y(k) &= \boldsymbol{\mathcal{B}}(q)\boldsymbol{\mathcal{A}}^{-1}(q)v(k) \qquad (1)\\ v(k) &= \mathbf{N}[u(k)] \ , \qquad (2) \end{aligned}$$

where $y(k)$ is the output vector and $u(k)$ is the input vector. Above, $\boldsymbol{\mathcal{B}}(q)$ is a stable and strictly proper rational matrix in the forward shift operator q, while $\boldsymbol{\mathcal{A}}(q)$ is a diagonal stable and biproper rational matrix with diagonal elements $\mathcal{A}_j$.[2] The memoryless nonlinearity $\mathbf{N}[\cdot]$ will in the following represent a bank of (possibly) saturating elements[3]

$$v_i(k) = \begin{cases} v_{i\ max} & \text{if} \quad u_i(k) > v_{i\ max} \\ u_i(k) & \text{if} \quad v_{i\ min} \leq u_i(k) \leq v_{i\ max} \\ v_{i\ min} & \text{if} \quad u_i(k) < v_{i\ min} \end{cases} \qquad (3)$$

where $i = 1 \ldots m$. A more compact notation for (3) is

$$v(k) = \text{sat}[u(k)] \quad . \qquad (4)$$

As a starting point of our discussion, a *nominal* controller

$$\boldsymbol{\mathcal{R}}u(k) = -\boldsymbol{\mathcal{S}}y(k) + \boldsymbol{\mathcal{T}}r(k) \qquad (5)$$

is assumed to be present. It is designed to fulfill appropriate specifications of an idealized linear closed loop, defined by connecting (5) to (1) with $v(k) = u(k)$, as depicted in Figure 1. Above, $\boldsymbol{\mathcal{R}}$, $\boldsymbol{\mathcal{S}}$, $\boldsymbol{\mathcal{T}}$ are stable and proper rational matrices in q, of appropriate dimension. We assume $\mathbf{I}_m - \boldsymbol{\mathcal{R}}$ to be strictly proper, and the idealized closed loop system to be stable.

In a state-space design, the expression (5) may, for example, constitute an input-output representation of an observer-based state feedback. The poles of $\boldsymbol{\mathcal{R}}$ and $\boldsymbol{\mathcal{S}}$ will then correspond to the observer dynamics. The controller may also have been designed in a polynomial framework, in which case $\boldsymbol{\mathcal{R}}$, $\boldsymbol{\mathcal{S}}$ and $\boldsymbol{\mathcal{T}}$ would correspond to polynomial matrices in the backward shift operator q^{-1}. (Polynomial matrices in z^{-1} are special cases of rational matrices in z, with all elements having all poles at the origin $z = 0$.)

Some consequences of input saturation, such as the inability to reach certain output values, will be unavoidable. Other effects, in particular limit cycles, instability, undesirable desaturation transients and undesirable changes in the direction of the control vector can be counteracted, if the saturated control vector $v(k)$ can be measured or reconstructed. Therefore we shall in the sequal assume that $v(k)$ can be measured or estimated without errors. It can then be utilized to modify the nominal control law (5). The most straightforward modification is to substitute saturated control signals $v(k)$ for all old values of $u(k)$ in the recursions

$$u(k) = (\mathbf{I}_m - \boldsymbol{\mathcal{R}})v(k) - \boldsymbol{\mathcal{S}}y(k) + \boldsymbol{\mathcal{T}}r(k) \quad . \qquad (6)$$

The structure (6) can be obtained by using saturated control signals in an observer. The modification (6) is often, but far from always, adequate. To obtain a satisfactory behaviour when inputs saturate, we may have to modify the controller dynamics, perhaps by de-tuning it so severely that the specifications can no longer be fulfilled in the nominal case. The controller (6) has no parameters that affect the behaviour caused by saturation, while leaving the dynamics unchanged when no components of $u(k)$ saturate. A systematic modification of the effects of actuator saturation becomes difficult unless additional degrees of freedom are introduced.

It can be shown that the most general realizable LTI controller, which reduces to the nominal control law (5) when no control element saturates, can be expressed as

$$u_W(k) = (\mathbf{I}_m - \boldsymbol{\mathcal{W}}\boldsymbol{\mathcal{R}})v(k) + \boldsymbol{\mathcal{W}}(\boldsymbol{\mathcal{T}}r(k) - \boldsymbol{\mathcal{S}}y(k)) \quad . \qquad (7)$$

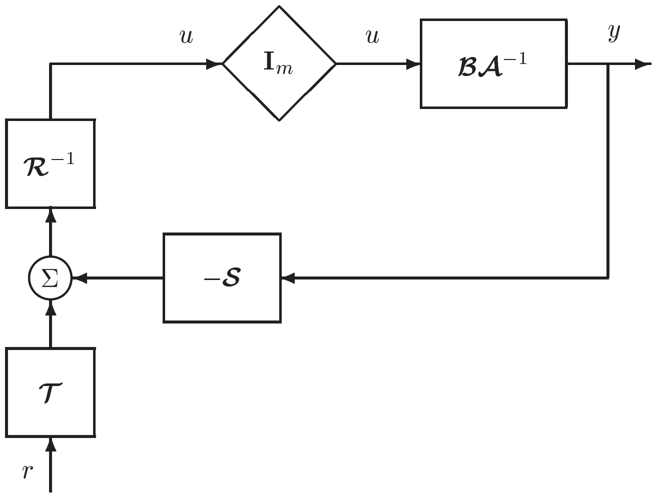

Figure 1: The process $\boldsymbol{\mathcal{B}}(q)\boldsymbol{\mathcal{A}}^{-1}(q)$ in rational fractional form, in closed loop with the nominal two degree of freedom controller. The closed loop is assumed stable.

[1] The anti-windup concept presented here is, however, applicable to both discrete time and continuous time systems.

[2] The argument q will be omitted here and in the follwing, if no risk of misunderstanding exists.

[3] The controller structure introduced in the present section can be utilized also in problems other than windup control, for example bumpless transfer. In that situation, $\mathbf{N}[\cdot]$ would represent swiches, possibly in series with saturation elements.

Here, $\boldsymbol{\mathcal{W}}$ is a stable and proper rational $m|m$ matrix, for which a stable and proper inverse $\boldsymbol{\mathcal{W}}^{-1}$ exists. Algebraic loops around the saturation element (4) must not be introduced. Conditions for the avoidance of algebraic loops are discussed in Appendix A. These conditions are fulfilled if $(\mathbf{I}_m - \boldsymbol{\mathcal{W}})$ is strictly proper, which will be assumed in the following.

Controlling the plant (1),(4) by use of (7) results in a closed-loop system depictured in Figure 2. In the nominal case, when $\mathrm{sat}[u_W(k)] = u_W(k)$, the closed loop system (1),(4),(7) becomes

$$y(k) = \boldsymbol{\mathcal{B}}(\boldsymbol{\mathcal{R}}\boldsymbol{\mathcal{A}} + \boldsymbol{\mathcal{S}}\boldsymbol{\mathcal{B}})^{-1}\boldsymbol{\mathcal{W}}^{-1}\boldsymbol{\mathcal{W}}\ \boldsymbol{\mathcal{T}}r(k) \ , \tag{8}$$

which corresponds to the nominal closed loop when $\boldsymbol{\mathcal{W}}^{-1}\boldsymbol{\mathcal{W}}$ is cancelled.

The rational matrix $\boldsymbol{\mathcal{W}}$ can be utilized as an *anti-windup filter*, which affects the linear dynamics during saturation events and also the transients which occur when control signals return to the linear range.

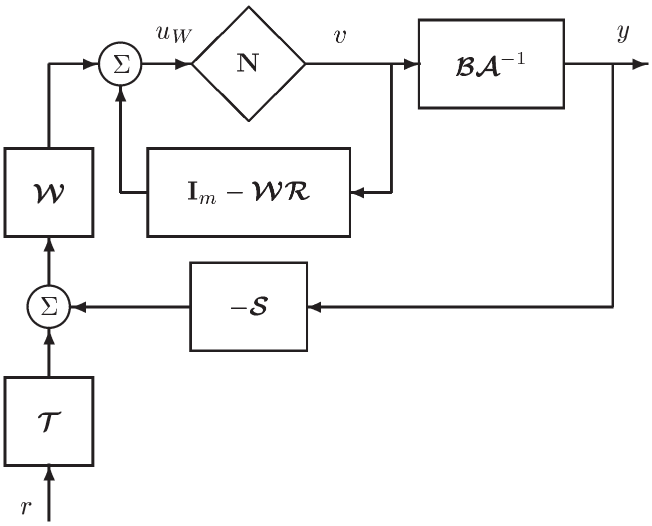

Figure 2: The process $\boldsymbol{\mathcal{B}}(q)\boldsymbol{\mathcal{A}}^{-1}(q)$ controlled in closed loop with a three degree of freedom controller structure. The anti-windup compensator $\boldsymbol{\mathcal{W}}(q)$ represents the third degree of freedom.

The controller structure (7) can be shown to be equivalent to the one discussed by Kothare *et al.* in [3][4]. An equivalent structure for polynomial designs in the scalar case was proposed by Rönnbäck *et al.* in [5].

Remark: As discussed in [3] and [6], the expression (7) includes the more restrictive controller structures utilized by previously suggested anti-windup schemes. Consider, for example, a controller for the scalar case designed by polynomial methods. The nominal controller is then represented by

$$\boldsymbol{\mathcal{R}} = \frac{R}{q^{nr}} \ , \ \boldsymbol{\mathcal{S}} = \frac{S}{q^{nr}} \ , \ \boldsymbol{\mathcal{T}} = \frac{T}{q^{nr}} \tag{9}$$

[4]The paper [3] mostly discusses the case of control error feedback, which corresponds to $\boldsymbol{\mathcal{T}} = \boldsymbol{\mathcal{S}}$ in (7).

where R, S and T are polynomials in q, all of degee nr [10]. The general scalar stucture $\boldsymbol{\mathcal{W}} = P/F$, where P and F are adjustable stable polynomials, was proposed by Rönnbäck *et al.* in [5]. The special case $\boldsymbol{\mathcal{W}} = q^{nr}/A_0$, where A_0 is an adjustable stable polynomial of degree nr, corresponds to the ***saturation observer-based*** anti-windup controller suggested by Åström and Wittenmark in [10]. The choice $\mathrm{A}_0 = T/t_0$, where t_0 is the leading coefficient of T in (9) will futhermore correspond to the *Conditioning technique* suggested by Hanus *et al.* in [2]. The properties of these and other strategies have been compared and discussed in, for example, [3, 6] and [7].

An existing controller with a simple saturation feedback described by (6) can be generalized to the form (7) without altering its internal structure. One can just insert a filter between the control signal generated by (6) and the nonlinearity (2).[5] The additional filtering to be performed is specified by

$$u_W(k) = (\mathbf{I}_m - \boldsymbol{\mathcal{W}})v(k) + \boldsymbol{\mathcal{W}}u(k) \ , \tag{10}$$

where $u(k)$ is obtained from (6).[6]

3 Tools for analysis

The main aim of anti-windup design, as it will be presented here, is to obtain a good transient behaviour after desaturation yet preserve stability. With a good transient behaviour we mean that

1. desaturation transients should have a fast decay;
2. limit cycles should not occur and repeated saturations should be avoided.

It is hard to address these issues with presently available methods. Several methods, such as the conditioning technique of [2], lack adjustable parameters. Others, such as the observer-based method of [10] do have adjustable parameters, but there exist no tools for adjusting them in a systematic way.

We will here utilize the linear approximation introduced by Rönnbäck in [6] when discussing and analyzing the linear loops around the bank of saturations. The difference between the actual and the saturated control signal is regarded as an input disturbance

$$\delta(k) \ \stackrel{\Delta}{=} \ v(k) - u_W(k) \ . \tag{11}$$

[5]This fact is important from a practical point of view. It may futhermore decrease the barrier of acceptance towards the use of model-based anti-windup schemes in general.

[6]The equivalence between a combination (6)(10) and the expression (7) is simple to demonstrate. Note that since $u_W(k)$ is used as the control signal, the signal $v(k)$ will equal $\mathrm{sat}[u_W(k)]$ in both (6) and (10).

By omitting the argument q, and combining (1),(4),(7) and (11), the closed-loop system is then obtained as

$$
\begin{aligned}
y(k) &= y_{nom}(k) + y_\delta(k) \\
&= \boldsymbol{\mathcal{H}}_{nom} r(k) + \boldsymbol{\mathcal{H}}_\delta \delta(k)
\end{aligned}
\tag{12}
$$

where

$$
\begin{aligned}
\boldsymbol{\mathcal{H}}_{nom} &= \boldsymbol{\mathcal{B}}(\boldsymbol{\mathcal{R}}\boldsymbol{\mathcal{A}} + \boldsymbol{\mathcal{S}}\boldsymbol{\mathcal{B}})^{-1}\boldsymbol{\mathcal{T}} \qquad (13)\\
\boldsymbol{\mathcal{H}}_\delta &= \boldsymbol{\mathcal{B}}(\boldsymbol{\mathcal{R}}\boldsymbol{\mathcal{A}} + \boldsymbol{\mathcal{S}}\boldsymbol{\mathcal{B}})^{-1}\boldsymbol{\mathcal{W}}^{-1} \quad . \qquad (14)
\end{aligned}
$$

Above $\boldsymbol{\mathcal{H}}_{nom}$ constitutes the nominal closed-loop system[7]. When a control signal exits saturation, the corresponding column of $\boldsymbol{\mathcal{H}}_\delta$ will determine the resulting transient behaviour. This simple linear formulation is enlightening when evaluating desaturation transients and other properties. It is, however, not a complete and adequate description of the closed loop; when some of the control signals saturate, $\delta(k)$ will be determined by a nonlinear feedback. We therefore have to introduce additional tools for controlling the nonlinear properties of the closed loop.

By inserting (1) into (7), the loop gain around the nonlinearity (4) can be expressed as

$$
\boldsymbol{\mathcal{L}}_v \triangleq \boldsymbol{\mathcal{W}}(\boldsymbol{\mathcal{R}}\boldsymbol{\mathcal{A}} + \boldsymbol{\mathcal{S}}\boldsymbol{\mathcal{B}})\boldsymbol{\mathcal{A}}^{-1} - \mathbf{I}_m \quad . \tag{15}
$$

Stability of the closed loop can be ascertained by studying the properties of the loop gain. The use of passivity-based methods, as suggested by e.g. Kothare and Morari in [4] is one possibility. This class of methods is, however, rather cumbersome to use, and in general results in conservative sufficient conditions for stability.

If $\boldsymbol{\mathcal{W}}$ is selected so that the loop gain becomes diagonal,

$$
\boldsymbol{\mathcal{L}}_v = \text{diag}(\mathcal{L}_j) \quad , \tag{16}
$$

there exists a simpler and less conservative alternative. Since the loops around the saturation elements are then decoupled, we may utilize scalar tools, in particular the *describing function*. When using the describing function $Y(C)$, it should be ascertained that the loop gain $\mathcal{L}_j(e^{i\omega})$ stays well away from the function $-1/Y(C)$. If the two functions intersect, then a limit cycle oscillation may be excited. If the loop gain comes close to but does not cross the inverse describing function, then repeated re-saturations occur when the signals leaves saturation. A safety margin must therefore be introduced between the Nyquist curve of the loop gain and the inverse describing function. see Figure 3.[8]

A discussion of the use of signal-dependent safety margins around the loop gain can be found in [7]. The use of a fixed avoidance sector, as suggested by Wurmthaler and Hippe in [9] and depicted in Figure 3, is mostly an adequate tool

[7]We have here cancelled the factor $\boldsymbol{\mathcal{W}}^{-1}\boldsymbol{\mathcal{W}}$ in (8) to obtain (13).

[8]Although we here illustrate the case of simple input saturation, the metod can be used for $\mathbf{N}[\cdot]$ being a diagonal matrix of arbitrary time-invariant memoryless nonlinearities.

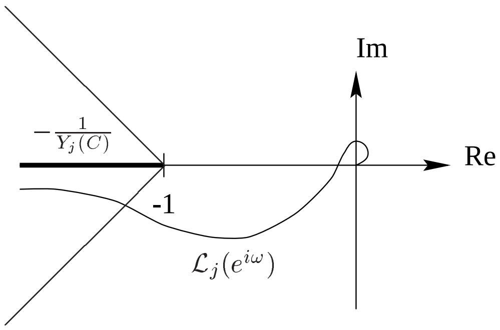

Figure 3: The scalar loop gain $\mathcal{L}_j(e^{i\omega})$ around actuator j and the inverse describing function, $-1/Y_j(C)$, of the nonlinear actuator. A safety sector which is to be avoided by the loop gain, is also shown. (In this particular case the loop gain enters the avoidance sector and hence, repeated re-saturations may occur.)

for predicting the ocurrence of repeated re-saturations of the actuators.

The transfer matrices $\boldsymbol{\mathcal{H}}_\delta$ from (14) and $\boldsymbol{\mathcal{L}}_v$ from (15) constitute our tools for investigating desaturation performance and stability. They will now both be utilized in the design of the anti-windup filter $\boldsymbol{\mathcal{W}}$.

4 Systematic anti-windup design

According to requirement 1. in Section 3, the dynamics of $\boldsymbol{\mathcal{H}}_\delta$ should be fast. This can be achieved by appropriate choices of $\boldsymbol{\mathcal{W}}$. However, if the dynamics of $\boldsymbol{\mathcal{H}}_\delta$ is made too fast, then repeated saturations and limit cycles may occur. Thus, the requirements 1. and 2. in Section 3 are often contradictory. It is therefore essential that an anti-windup design includes a trade-off between a fast transient and a small influence of nonlinear effects. A design method is presented next, which utilizes simple scalar tools for attaining such a trade-off.

For the design of the anti-windup filter $\boldsymbol{\mathcal{W}}$ in (7), the criterion

$$
J = \|\boldsymbol{\mathcal{H}}_\delta\|_2^2 + \left\| \mathbf{Q}\left((\boldsymbol{\mathcal{L}}_v + \mathbf{I}_m)^{-1} - \mathbf{I}_m \right) \right\|_2^2 \tag{17}
$$

is introduced. This criterion was suggested for the scalar case by Sternad and Rönnbäck in [7]. In (17), $\mathbf{Q}$ is a diagonal penalty matrix. In the limiting case $\mathbf{Q} = 0$, minimization of J corresponds to the minimization of the $\boldsymbol{\mathcal{H}}_2$ norm of the stable transfer matrix $\boldsymbol{\mathcal{H}}_\delta$. The second term of J penalizes loop gains which may cause instability. In particular, it penalizes loop gains $\boldsymbol{\mathcal{L}}_v$ which come close to $-\mathbf{I}_m$. Now, by choosing $\boldsymbol{\mathcal{W}}$ as

$$
\boldsymbol{\mathcal{W}} = \boldsymbol{\mathcal{P}}(\boldsymbol{\mathcal{R}}\boldsymbol{\mathcal{A}} + \boldsymbol{\mathcal{S}}\boldsymbol{\mathcal{B}})^{-1} \quad , \tag{18}
$$

where $\boldsymbol{\mathcal{P}}$ is a rational matrix to be determined, the rational matrices $\mathcal{H}_\delta$ and $\mathcal{L}_v$ reduce to

$$\mathcal{H}_\delta = \mathcal{B}\mathcal{P}^{-1} \ ; \ \mathcal{L}_v = \mathcal{P}\mathcal{A}^{-1} - \mathbf{I}_m \ , \tag{19}$$

respectively. We restrict both $\mathcal{P}$ and $\mathcal{P}^{-1}$ to be stable and proper. Since $\mathcal{A} = \text{diag}(\mathcal{A}_j)$, selecting $\mathcal{P}$ *diagonal* makes $\mathcal{L}_v$ become diagonal.

By insertion of (19) into (17), the criterion can be rewritten in the form

$$J = \left\|\mathcal{B}\mathcal{P}^{-1}\right\|_2^2 + \left\|\mathbf{Q}\left(\mathcal{A}\mathcal{P}^{-1} - \mathbf{I}_m\right)\right\|_2^2 \ . \tag{20}$$

Let $\mathcal{B}_{ij}$ be the ij-th scalar rational element of $\mathcal{B}$ in (1). The minimum of (20), with respect to a diagonal $\mathcal{P}$, for a given diagonal penalty matrix $\mathbf{Q} = \text{diag}\sqrt{\rho_j}$, is shown in [11] to be attained by solving m separate scalar spectral factorization equations

$$r_j\mathcal{P}_j\mathcal{P}_j^* = \sum_{i=1}^{p}\mathcal{B}_{ij}\mathcal{B}_{ij}^* + \rho_j\mathcal{A}_j\mathcal{A}_j^* \tag{21}$$

$$\mathcal{P} = \text{diag}(\mathcal{P}_j) \ . \tag{22}$$

Here,r_j is a scale factor. Equation (21) has to be solved for $j = 1, 2....m$, where m is the number of process inputs, p is the number of process outputs, The design of a multivariable anti-windup compensator is thus reduced to m scalar designs, in which the m elements of the diagonal loop gain matrix $\mathcal{L}_v$ are systematically adjusted.

Note that if $\rho_j \to \infty$, then $\mathcal{P}_j \to \mathcal{A}_j$. The jth loop gain $\mathcal{L}_j$ will then contract and stay well away from the negative real axis, if $\mathcal{A}_j$ is stable[9]. As a result, repeated saturations will not occur in that loop. However, the desaturation transients may then show an unsatisfactory behaviour, since the common denominator of the jth column of $\mathcal{H}_\delta$ goes towards the plant dynamics $\mathcal{A}_j$, which may be slow of oscillative.

On the other hand if ρ_j is selected small, the dynamics of the jth column of $\mathcal{H}_\delta$ will become fast, while the jth loop gain may become large, and come close to the negative real axis to the left of -1. This, in turn, may generate repeated saturations and limit cycles. The user must therefore select the values of ρ_j properly to obtain an appropriate trade-off.

5 Simulation example

The model used for simulation describes a *Heavy Oil Fractionator* [1], with two inputs and two outputs. The controller in (7) is used with two different choices of $\mathcal{W}$. In both the cases, the nominal controller, $\mathcal{R}$, $\mathcal{S}$, $\mathcal{T}$, originates from an observer-based state-feedback LQ-control law, expressed in input-output form. The model used for controller- and anti-windup filter design is obtained by subspace-identification.

In the first case, we select $\mathcal{W} = \mathbf{I}_m$, which simply means that the observer is fed with saturated control signals, as in (6). The two resulting outputs from the simulation are shown in the upper pair of diagrams in Figure 5, and the two corresponding control signals are shown in the third pair of diagrams from the top. These four diagrams are all labeled with (W=I).

In the second case, the method proposed in this paper was used for the design of $\mathcal{W}$. The result, after adjustment of the penalties ρ_1, ρ_2, is shown in the second pair of diagrams from the top, and in the two bottom diagrams, all labeled (W optimal).

In this example, it is clearly worthwile to optimize the anti-windup filter $\mathcal{W}$.

A Realization

Due to the fact that $v(k) = sat[u_W(k)]$, care must be taken in the realization of the control laws (6)(7)(10) to avoid algebraic loops. The key property of relevant rational matrices $\mathcal{M}(q)$ is their leading coefficient matrix $\mathbf{M}_0$ in a series expansion in the backward shift operator q^{-1}, i.e. in the puls response

$$\mathcal{M} = \mathbf{M}_0 + \mathbf{M}_1 q^{-1} + \dots \ . \tag{23}$$

Consider a feedback connection of a memoryless nonlinearity $v(k) = \mathbf{N}[u(k)]$ and the system $u(k) = \mathcal{M}(q)v(k)$. If arbitrary cross-connections are allowed in $\mathbf{N}[\cdot]$, then the absence of algebraic loops is guaranteed if and only if $\mathbf{M}_0 = 0$.[10] As an example, consider $\mathcal{M} = \mathbf{I}_m - \mathcal{W}\mathcal{R}$ from (7). Since $\mathbf{I}_m - \mathcal{R}$ is stricly proper by assuption, we have $\mathcal{R} = \mathbf{I}_m + \mathbf{R}_1 q^{-1} + \dots$. If $\mathbf{I}_m - \mathcal{W}$ is restricted to be stricly proper, then $\mathcal{W} = \mathbf{I}_m + \mathbf{W}_1 q^{-1} + \dots$. Thus, $\mathbf{M}_0 = 0$, so no algebraic loops will occur.

The filter (10) can be realized in the following way, with $u_W(k) = u_{w1}(k) + u_{w2}(k)$: If $u_{w1}(k) \triangleq \mathcal{W}u(k)$ is given by a state space-realization

$$\begin{aligned} x_1(k+1) &= \mathbf{A}x_1(k) + \mathbf{B}u(k) \\ u_{w1}(k) &= \mathbf{C}x_1(k) + \mathbf{D}u(k) \ , \quad \mathbf{D} = \mathbf{I}_m \end{aligned} \tag{24}$$

then $u_{w2}(k) \triangleq (\mathbf{I}_m - \mathcal{W})v(k)$ is given by

$$\begin{aligned} x_2(k+1) &= \mathbf{A}x_2(k) + \mathbf{B}v(k) \\ u_{w2}(k) &= -\mathbf{C}x_2(k) \ . \end{aligned} \tag{25}$$

[9] For stable $\mathcal{A}_j$, the loop gain $\mathcal{L}_j$ vanishes when $\rho_j \to \infty$. It is mostly possible to find adjustments of ρ_j which push $\mathcal{L}_j$ outside the avoidance sector indicated in Figure 3, but there are exceptions. If triple or higher order integrators are present in $\mathcal{A}_|$, then a crossing between the loop gain and the describing function cannot be avoived. The intersection point must then be placed so far to the left that no disturbances with reasonable amplitudes will excite limit cycle oscillations.

[10] If $\mathbf{N}[\cdot]$ is diagonal, as in (4), then the interconnection can be realized without algebraic loops if $\mathbf{M}_0$ is either strictly upper or lower triangular, with only zeros on the diagonal.

Let us now briefly discuss one possible way to find a state space-realization for the filter $\mathcal{W}$, when it is given by the expression $\mathcal{W} = \mathcal{P}(\mathcal{R}\mathcal{A} + \mathcal{S}\mathcal{B})^{-1}$. Since the matrix $\mathcal{P}$ has a stable inverse, the filter $\mathcal{W}$ can be rewritten as

$$\mathcal{W} = (\mathcal{R}\mathcal{A}\mathcal{P}^{-1} + \mathcal{S}\mathcal{B}\mathcal{P}^{-1})^{-1} \tag{26}$$

This transfer function matrix can be obtained from a closed loop involving four filters:

$$\mathcal{P}\mathcal{A}^{-1}\ ,\ \mathcal{B}\mathcal{P}^{-1}\ ,\ \mathcal{R}^{-1}\ ,\ -\mathcal{S}\ , \tag{27}$$

which are rather straightforward to realize individually. See Figure 4. The final step is to combine these filters to obtain a state space-realization as the one in (24).

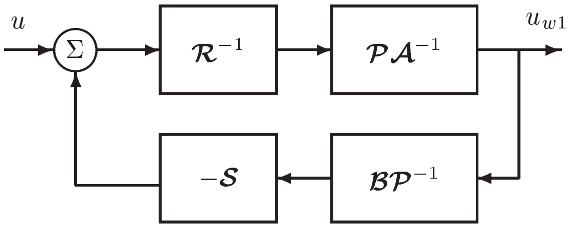

Figure 4: Closed loop representation of the filter $\mathcal{W} = \mathcal{P}(\mathcal{R}\mathcal{A} + \mathcal{S}\mathcal{B})^{-1}$.

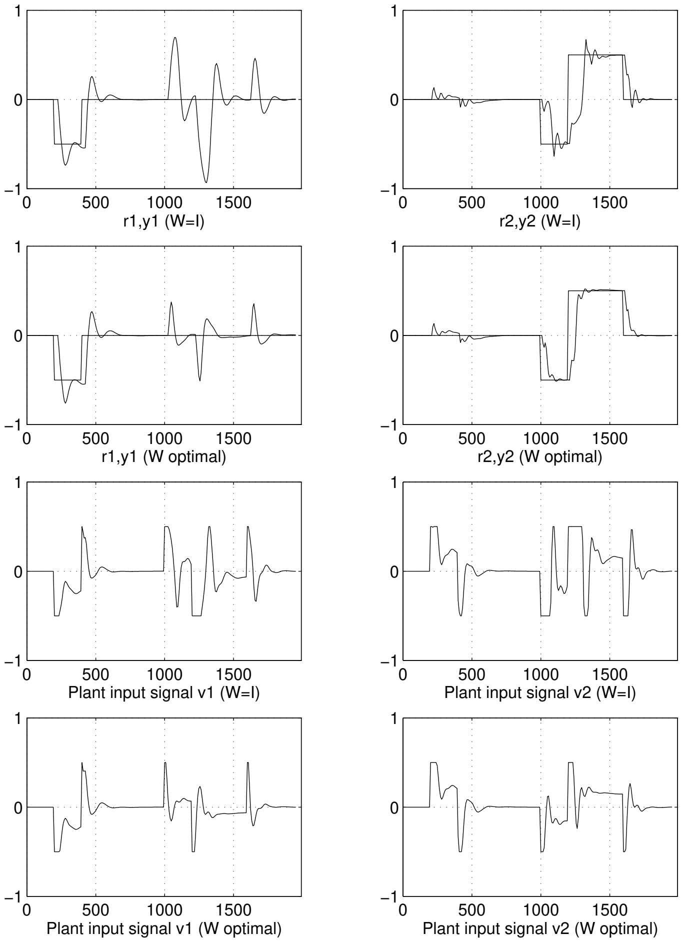

Figure 5: Simulation of a Heavy Oil Fractionator process controlled by an observer-based state-feedback control law. Two different anti-windup strategies are used.

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