Designing Equalizers Based on Explicit Channel Models of DS-CDMA systems

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ABSTRACT In this paper, the entire process of transmission and reception in a DS-CDMA system, including the spreading, is described as a tapped delay line model with multiple inputs and a single output. This model is then used to design a fractionally spaced decision feedback equalizer with a single input and multiple outputs. In the proposed approach, long codes can be used and channel estimation can be performed in an efficient way. The suggested detector is near-far resistant. Simulations demonstrate the utility of the proposed approach in a heavily loaded system with and without power control where multipath is present.

1 Introduction

Much of the flexibility in the design of a DS-CDMA system relies on the use of *long spreading codes*. Long spreading codes have a period that is much larger than the symbol period. If long codes are used, code planning is not necessary, and it is very easy to supply variable rate services.

The use of non-orthogonal channels makes the DS-CDMA system very sensitive to the *near-far problem*. The common solution to this problem, power control in combination with the conventional detector, does however have serious drawbacks. To circumvent the near-far problem and thus eliminate the need for power control, the use of so-called *multiuser detectors* has been proposed.

Most proposed multiuser detectors operate on the outputs from a bank of filters, matched to the spreading codes of each user. These multiuser detectors are frequently presented as block detectors in systems where multipath propagation and long codes are rarely considered explicitly. One example of such a multiuser detector is the decorrelating detector [1]. A disadvantage with these detectors is that the propagation delay must be well known. However, in a near-far scenario, the propagation delay can be difficult to estimate, as shown in [2].

Other multiuser detectors operate directly on the received wideband signal which is sampled at the chip rate [3, 4, 5]. The received signal is passed through an FIR filter followed by a decision device. The filters are adaptive and they are updated recursively directly from the received signals, making estimation of the propagation delay unnecessary. On the other hand, long codes are impossible to use, since the cyclostationarity in the signals is destroyed when the codes are not periodic with period equal to the symbol period. Also, low bit rates could result in a difficult tracking problem, since the optimal filter coefficients will change substantially between two updates in the adaptive algorithm.

An ideal detector for DS-CDMA should be near-far resistant like the decorrelating detector. It should also be insensitive to errors in propagation delay estimates like the adaptive detectors that operate directly on the chip sampled signal. Finally, the detector should be able to operate in a system where long codes are employed like the conventional detector.

In this paper, we reformulate the standard DS-CDMA system transmission and reception as an equivalent, discrete time, tapped delay line model. This model takes the data symbols as input and has the wideband, chip sampled, signal as output. The spreading operation is represented as linear filtering, which is time-invariant for short codes and time-varying for long codes. This model makes it possible to design equalizers which, directly from the chip sampled signal, filter out the signals of the respective users.

As an example of a detector which has been derived from this model, we subsequently present a generalization of the fractionally spaced decision feedback equalizer (DFE). This DFE has one input and the same number of outputs as the number of users. We then use Monte Carlo simulations to investigate the performance of the proposed DFE with perfect power control, with average power control and without any power control. For a rather heavily loaded system, the DFE is shown to outperform the conventional detector with a RAKE receiver in the first two cases. In the third case, it is demonstrated that the DFE works without problem in a near-far situation.

2 System model

In this section a DS-CDMA system will be reformulated as a completely discrete time, tapped delay line model with multiple inputs and a single output.

2.1 The DS-CDMA system

We are considering an asynchronous DS-CDMA system with K users. The modulation scheme is BPSK. The processing gain, i.e. the ratio between the symbol period and the chip period, is denoted by N_c.

It is assumed that user k transmits the symbol $d_k(tN_c)$ during the time period $[tN_c, (t+1)N_c[$ (in units of chip periods). Each symbol is spread by a wideband signature sequence. The spreading operation results in a baseband signal, which is shifted up to the carrier frequency and transmitted.

The transmitted passband signal propagates through a frequency selective wideband channel. In the receiver front end the I and Q signals are down-converted to the baseband. The received baseband signal is then passed through a *chip-matched filter* and the resulting complex-valued signal will be denoted by $r(\cdot)$. In the following, only equivalent baseband signals and channels will be considered.

We will now describe how this standard DS-CDMA system can be recast into a form, in which the entire process of transmission and reception can be represented by a completely discrete time, tapped delay line model. This tapped delay line model will have the (narrowband) symbol sequence $d_k(tN_c)$ as input at time tN_c. The received (wideband) signal r will be the output of the tapped delay line. The impulse response of the model might be rapidly time-varying.

2.2 A linear baseband model: the single-user case

Traditionally, both symbol and signature sequences in a DS-CDMA system are presented as *continuous time signals*. To arrive at a discrete time model of the system, all pulse shaping is assumed to take place *after* the spreading. This means that the system can be represented as in Figure 1.

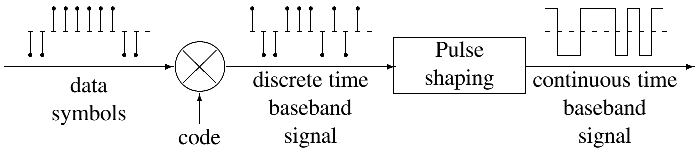


Figure 1 The spreading viewed as a discrete time process

Next, the pulse shaping, the frequency up-conversion, the physical channel, the frequency down-conversion and the chip-matched filter are lumped together and replaced by an equivalent discrete time channel. This equivalent representation will in the following be called the *physical channel*. This means that the transmission and reception can be represented as in Figure 2.

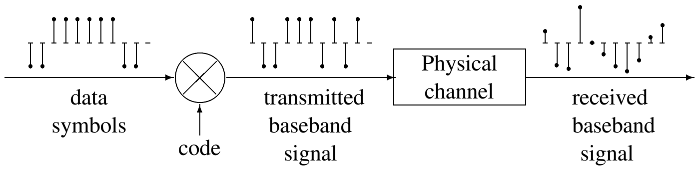


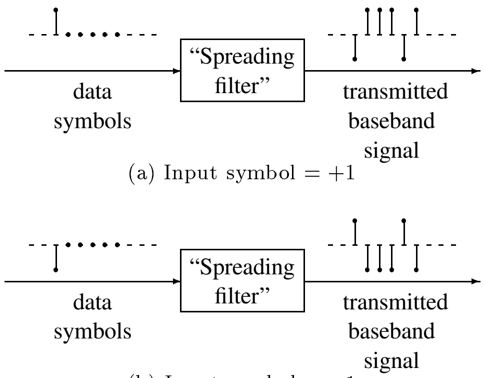
Figure 2 A completely discrete time model.

The model is now completely discrete time, but it is not yet a tapped delay line model. In order to arrive at such a model, the spreading must be represented by a linear filter. We would then obtain an *equivalent channel* which can be represented by a convolution of this *spreading filter* and the physical channel.

Let us therefore view the symbol sequence as an input and the spread signal as an output of such a linear filter. Consider the situation during one symbol interval. If the current symbol is $+1$, then the transmitted baseband signal will be the code, whereas if the current symbol is -1, the transmitted signal will be the code with opposite sign. The situation is depicted in Figure 3. This means that during one symbol period, *the spread signal can be thought of as having been generated by passing the symbol through a discrete time filter with impulse response identical to the code sequence.*[1] The length of the impulse response of the spreading filter is equal to the processing gain N_c.

The entire single-user channel from the symbol sequence to the chip sampled received baseband sequence can thus be

[1] For short codes, the spreading filter will be time-invariant, whereas for long codes it will change every symbol period.



(a) Input symbol $= +1$

(b) Input symbol $= -1$

Figure 3 The spreading viewed as a filtering operation.

represented by a tapped delay line. This tapped delay line is the convolution between the spreading filter and the physical channel. Since the length of the spreading filter is equal to the symbol period, intersymbol interference will be present if the physical channel has more than one tap.

2.3 A multiple-input single-output channel model

In a multiuser scenario with K users, each user transmits a symbol sequence through a separate tapped delay line. The received signal will be the sum of the outputs from these K channels and some noise. The entire (multiuser) channel can thus be represented by a linear model with K inputs and one output. To formulate this mathematically, denote the output of the channel from user k at time $tN_c + j$ as

$$z_k(tN_c + j) = \sum_{i=0}^{m} h^k_{iN_c+j}(tN_c + j) d_k((t-i)N_c) \quad , j = 0, 1, \ldots, N_c - 1 \qquad (1)$$

where

$h^k_{iN_c+j}(tN_c + j)$	:	tap $iN_c + j$ in the channel from user k at time $tN_c + j$
m	:	maximum extent (over all channels) of intersymbol interference.

The received signal will be the noise-corrupted sum of the signals from the K users:

$$r(tN_c + j) = \sum_{k=1}^{K} \sum_{i=0}^{m} h^k_{iN_c+j}(tN_c + j) d_k((t-i)N_c) + n(tN_c + j)$$

where $n(tN_c + j)$ is assumed to be wide sense stationary, zero mean noise with autocorrelation function

$$E\left[n(i)n^H(j)\right] \triangleq \psi_{i-j}$$

Introduce the vector of symbols transmitted by all users at time tN_c

$$d(tN_c) \triangleq \begin{pmatrix} d_1(tN_c) & \ldots & d_K(tN_c) \end{pmatrix}^T$$

and define

$$\mathbf{h}^{tN_c+j}_{iN_c+j} \triangleq \begin{pmatrix} h^1_{iN_c+j}(tN_c + j) & \ldots & h^K_{iN_c+j}(tN_c + j) \end{pmatrix} \qquad (2)$$

Then the received signal $r(tN_c + j)$ can be rewritten as

$$r(tN_c + j) = \sum_{i=0}^{m} \mathbf{h}_{iN_c+j}^{tN_c+j} d((t-i)N_c) + n(tN_c + j) \ . \qquad (3)$$

Equation (3) is the desired K-input single-output channel model, which relates the sequence of symbol vectors to the chip sampled output sequence.

To make use of this model for the design of a DS-CDMA detector, we first have to estimate the channel coefficient vector (2) for $i = 0, 1, \dots, m$ and $j = 0, 1, \dots, N_c - 1$. We then have to design a detector (an equalizer) under the assumption that the impulse response coefficients (2) are known. An equalizer is necessary, since $m \neq 0$ in general.

When performing the channel estimation, we suggest the use of the transmitted chip sequence as regressors to identify *only the physical channel*. The complete channel is then obtained by convolving the (estimated) physical channel with the (known) spreading filter.

One good candidate for an equalizer is the decision feedback equalizer (DFE). To be applicable in this context, a DFE has to be fractionally spaced, because the received (information bearing) signal has a bandwidth which is considerably larger than the symbol rate.

3 The single-input K-output fractionally spaced DFE

Our proposed decision feedback equalizer operates directly on the chip sampled signal $r(\cdot)$. The outputs from the equalizer are the data symbols of the K users. This means that the proposed DFE will be a single-input multiple output-equalizer.

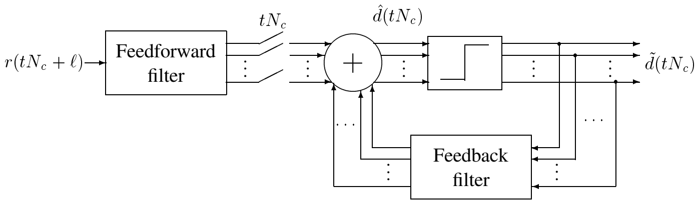


Figure 4 The single-input multiple-output fractionally spaced decision feedback equalizer.

The structure of the DFE is depicted in Figure 4. The scalar, chip sampled signal $r(\cdot)$ is used as input to the feedforward filter. The K outputs of the feedforward filter are sampled at the symbol rate. The influence of previously detected symbols are removed by subtracting the K outputs of the feedback filter from the corresponding K outputs of the feedforward filter. The resulting signal (vector) $\hat{d}(tN_c)$ is an estimate of the data symbols of the K users. This estimate is then passed through a decision device, where hard decisions are made.

The feedforward filter is specified by its coefficient matrices $\mathbf{S}_i^{tN_c}$ of dimension $K|1$, whereas the feedback filter is specified by its coefficient matrices $\mathbf{R}_i^{tN_c}$ of dimension $K|K$. The estimator can then be expressed as:

$$\hat{d}(tN_c|tN_c+\ell) = \sum_{i=0}^{\ell} \mathbf{S}_i^{tN_c} r(tN_c + \ell - i) - \sum_{i=1}^{m} \mathbf{R}_i^{tN_c} \tilde{d}((t-i)N_c) \ . \qquad (4)$$

The DFE estimates the symbol vector that was transmitted at time tN_c, given data up to time $tN_c + \ell$. The design variable ℓ is known as the *smoothing lag* or *decision delay.*

Introduce the parameter matrices

$$\Theta_S^{tN_c} \triangleq \begin{pmatrix} \mathbf{S}_0^{tN_c} & \dots & \mathbf{S}_\ell^{tN_c} \end{pmatrix}^H \qquad (5a)$$

$$\Theta_R^{tN_c} \triangleq \begin{pmatrix} \mathbf{R}_1^{tN_c} & \dots & \mathbf{R}_m^{tN_c} \end{pmatrix}^H \qquad (5b)$$

of dimensions $(\ell+1)|K$ and $mK|K$ respectively. We also define the matrices

$$\mathcal{F}_t \triangleq \begin{pmatrix} \bar{\beta}_0^{t+q} & & \dots & \bar{\beta}_q^{t+q} \\ 0 & \beta_0^{t+q-1} & \dots & \beta_{q-1}^{t+q-1} \\ \vdots & \ddots & \ddots & \vdots \\ 0 & \dots & 0 & \beta_0^t \end{pmatrix}$$

$$\mathcal{G}_t \triangleq \begin{pmatrix} \bar{\beta}_{q+1}^{t+q} & \dots & \bar{\beta}_m^{t+q} & 0 & \dots & 0 \\ \beta_q^{t+q-1} & \dots\dots & \beta_m^{t+q-1} & & \ddots & \vdots \\ \vdots & & & & \ddots & 0 \\ \beta_1^t & \dots & \dots & \dots & \dots & \beta_m^t \end{pmatrix}$$

where

$$\beta_i^t \triangleq \begin{pmatrix} \mathbf{h}_{iN_c+N_c-1}^{tN_c+N_c-1} \\ \mathbf{h}_{iN_c+N_c-2}^{tN_c+N_c-2} \\ \vdots \\ \mathbf{h}_{iN_c}^{tN_c} \end{pmatrix} \text{ and } \bar{\beta}_i^t \triangleq \begin{pmatrix} \mathbf{h}_{iN_c+p}^{tN_c+p} \\ \mathbf{h}_{iN_c+p-1}^{tN_c+p-1} \\ \vdots \\ \mathbf{h}_{iN_c}^{tN_c} \end{pmatrix} \quad i = 0, \dots, q \ ,$$

of dimensions $N_c|K$ and $(p+1)|K$ respectively. The non-negative integers p and q are implicitly defined by $\ell \triangleq qN_c + p$. For the calculation of the DFE, the following matrices are also required:

$$h_t \triangleq \begin{pmatrix} \mathbf{h}_\ell^{tN_c+\ell} \\ \mathbf{h}_{\ell-1}^{tN_c+\ell-1} \\ \vdots \\ \mathbf{h}_0^{tN_c} \end{pmatrix} \text{ and } \Psi \triangleq \begin{pmatrix} \psi_0 & \psi_1 & \dots & \psi_\ell \\ \psi_{-1} & \psi_0 & \ddots & \vdots \\ \vdots & \ddots & \ddots & \psi_1 \\ \psi_{-\ell} & \dots & \psi_{-1} & \psi_0 \end{pmatrix} .$$

The following result describes how the matrices (5a) and (5b) are calculated:

Theorem. Consider the time-varying fractionally spaced DFE described by (4) and the time-varying channel model described by (3). Assuming correct past decisions, the matrix coefficients $\mathbf{S}_i^{tN_c}$ and $\mathbf{R}_i^{tN_c}$ that minimize the MSE of the symbol estimate $\hat{d}(tN_c|tN_c+\ell)$ are obtained as follows:

1. Solve the system of linear equations

$$\left(\mathcal{F}_t \mathcal{F}_t^H + \Psi\right) \Theta_S^{tN_c} = h_t \qquad (6)$$

for the matrix coefficients $\mathbf{S}_i^{tN_c}$.

2. Perform the matrix multiplication

$$\Theta_R^{tN_c} = \mathcal{G}_t^H \Theta_S^{tN_c} \tag{7}$$

to obtain the matrix coefficients $\mathbf{R}_i^{tN_c}$.

Proof: See [6]. ■

Remark 1 The time-varying nature of the channel is explicitly taken into account by including *future* channel models in the calculation of the optimal filter coefficients. If long codes are used, the channel will in fact change completely every symbol period.

Remark 2 Unlike most multiuser detectors, the proposed DFE is a symbol-by-symbol detector, making it possible to implement it without modification.

The computational complexity of the calculations necessary to optimize the equalizer coefficients is rather high. To calculate the coefficients of the feedforward filter, approximately $(\ell+1)^3/6 + K(\ell+1)^2$ complex multiplications have to be performed. The number of multiplications necessary to calculate the coefficients of the feedback filter is proportional to mK^2. To actually estimate a transmitted symbol vector then requires $(\ell+1)K + mK^2$ complex multiplications. Notice that the complexity is not exponential in any system parameter.

4 Simulations

4.1 Simulation conditions

We will now investigate the utility of our channel model, as well as the usefulness of our proposed DFE with the aid of Monte Carlo simulations. We are using *long* codes in a system with processing gain $N_c = 8$ and $K = 5$ users. This represents a rather heavily loaded system.

All physical channels have four Rayleigh faded taps. All taps fade independently, but during transmission of a burst, they are assumed time invariant. The propagation delays of the users are uniformly distributed in the interval $[0, N_c - 1]$ (in units of the chip period). Only delays that are integer multiples of the chip period are considered. The additive noise is white and Gaussian. In the simulation study we assume that we know the channel impulse response, as well as the propagation delays and the color of the noise.

For each SNR, 1000 channels were randomly selected. Over each channel, each user transmitted 1000 symbols, which were subsequently detected.

We are considering three power control scenarios: perfect power control, average power control (explained in Section 4.3) and no power control. The results of the simulations are presented next.

4.2 Perfect power control

With perfect power control, the single-input K-output fractionally spaced DFE is compared to the conventional detector with a four-finger RAKE receiver. The E_b/N_0, which is identical for the five users, varies between 5 and 20 dB and the average BER of the users is estimated.

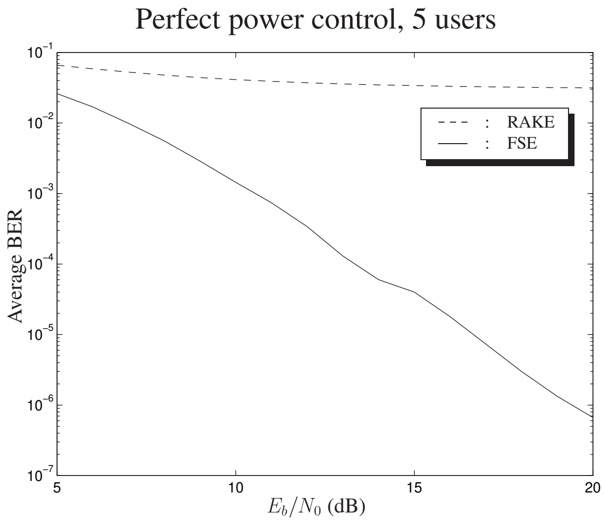


Figure 5 Average BER as a function of E_b/N_0 for DFE (solid) and conventional detector with RAKE (dashed). Perfect power control is used.

The results are shown in Figure 5. The RAKE receiver is clearly inferior to the DFE at all investigated signal-to-noise-ratios. This is natural, since the codes are not orthogonal at the receiver. The interference limited nature of the conventional receiver is clearly visible: the BER does not decrease much as E_b/N_0 increases.

The DFE on the other hand is not limited by the interference from the other users. The BER just decreases as the SNR is increased. The influence from other users is removed by the DFE, which decorrelates the signals. The performance of the DFE is then limited, not by interference from other users within the same cell, but by the interference from users in other cells, which will have to be considered as (colored) noise.

4.3 Average power control

The demand of strict power control is abandoned in this simulation setup. We are only assuming that the received signals have the same *average* power. This means that the power control will not have to compensate for the fast fading, but only for the shadowing loss. Apart from this, the simulation conditions are the same as in Section 4.2.

The results are shown in Figure 6. Both the RAKE receiver and the DFE perform worse in this case, as compared to the results of Figure 5. The BER for the RAKE is about 50% higher for the entire range of SNR:s compared to the case with perfect power control. The BER for the DFE is about twice as high as it was for the case of perfect power control for low SNR:s. For high SNR:s, the BER is ten times higher. The important property of the DFE being a noise limited, rather than an interference limited, detector is however preserved. Still, there is no visible error floor.

4.4 No power control

To investigate the performance of the DFE in a situation where the near-far problem is present, we have performed a simulation where not even the average power of the users are

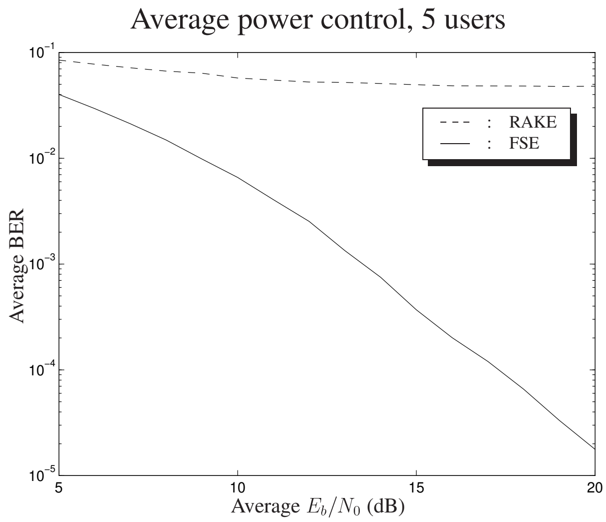


Figure 6 Average BER as a function of average E_b/N_0 for DFE (solid) and conventional detector with RAKE (dashed). Average power control is used.

equal. To emphasize the near-far problem, four of the users have the same average power, whereas the fifth user has an average power that is 10, 20 or 30 dB lower. The weaker user (user 1) has an average E_b/N_0 between 5 and 20 dB.

The performance measure in this case is the BER of the weaker user, because the number of errors for the stronger users will not be noticeable. Since the conventional receiver cannot deal with large near-far effects, the RAKE receiver was not included in this simulation scenario.

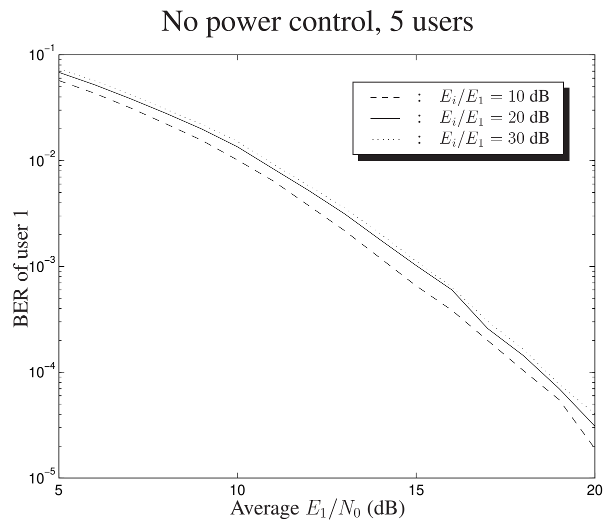


Figure 7 Average BER as a function of average E_b/N_0 for the weaker user (user 1). No power control is used.

The result is shown in Figure 7. We see that the near-far effect has an impact on the DFE, but the effect is small: for a power difference of 10 dB, the deterioration is around 1 dB for the entire investigated range of SNR:s as compared to average power control.

5 Conclusions

In this paper, we have described the transmission and reception process in a DS-CDMA system, including the spreading, by a discrete time tapped delay line model. Inputs to the model are the symbols from all users in the system. The output of the model is the received chip-sampled signal. By using this discrete time model, the multiuser detection problem has been recast as a multivariable equalization problem. Also, the reformulation makes it possible to perform channel estimation in a way that reduces problems with fast fading.

To demonstrate the usefulness of the reformulation, we have subsequently derived a fractionally spaced decision feedback equalizer based on the proposed model. This DFE detects all signals simultaneously, thereby making it possible to operate in an environment without power control. In contrast to many other multiuser detectors, it is derived as a symbol-by-symbol detector, making it well suited for implementation. Due to the fast sampling, the DFE is not sensitive to inaccurate synchronization.

The DFE can be used in systems where long codes are employed. Since the DFE effectively decorrelates the users, a large part of the capacity loss commonly associated with such codes could be eliminated.

The simulations in Chapter 4 reveal that the DFE outperforms the conventional detector in a heavily loaded system when perfect or average power control is used. It is also demonstrated that the DFE works well in the same system but with a severe near-far problem.

The amount of computations that has to be performed to update the equalizer is substantial, but it is not exponential in any system parameter.

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