Systematic Anti-Windup Compensator Design for Multivariable Systems*

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Abstract

The aim of anti-windup compensation is to modify the dynamics of a control loop when control signals saturate, so that a good transient behaviour is attained after desaturation, while avoiding nonlinear oscillations and repeated saturations.

Model-based anti-windup compensation is here considered for multiple-input multiple-output (MIMO) systems. A modified controller structure is proposed, which leaves the nominal closed-loop dynamics unchanged, as long as none of the control signals saturate. The proposed approach is applicable to continuous-time as well as discrete-time systems. Although it is developed for systems in input-output form, it can be used for systems in state-space form as well.

1 Introduction

The problem of finding controllers which have desired properties during or after saturation events has, over the years, resulted in a number of different anti-windup strategies. Many of the proposed methods focus on adjusting the states of the controller during a saturation event. For reasons explained in [6], and more recently also in [3, 4], there is, however, no guarantee that the *whole* system behaves acceptably during or after a saturation event, when only *controller*-state windup is prevented. Repeated saturations and even limit cycles might occur. To avoid such effects, the whole linear dynamics around the saturating elements, consisting of nominal controller elements, anti-windup filters and the open-loop plant, has to be taken into account. In the scalar case this can be accomplished in a Nyquist diagram: the loop gain around the saturating element is adjusted so that it stays well away from the function $-1/Y(C)$, where $Y(C)$ is the describing function of the saturation nonlinearity. The aim of the present paper is to introduce a controller structure which makes it possible to generalize this technique for analysis and design to feedback systems with multiple control signals. A key simplification is that the loop gain

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relevant for the saturation behaviour is made diagonal. The diagonal elements can then be adjusted in the same way as for a scalar system.

2 Problem formulation

The aim of anti-windup design, as it will be presented here, is to obtain a good transient behaviour after desaturation. With a good transient behaviour we mean that

1. desaturation transients should have a fast decay;
2. limit cycles and repeated saturations should not occur.

To achieve these aims, a method developed in [8] will be utilized.

Consider a discrete-time[1] multivariable, stable or marginally stable system with m inputs and p outputs, parameterized in rational fractional form as

$$\begin{aligned} y(k) &= \boldsymbol{\mathcal{B}}(q)\boldsymbol{\mathcal{A}}^{-1}(q)v(k) \\ v(k) &= sat[u_W(k)] \quad . \end{aligned} \tag{1}$$

Above, $\boldsymbol{\mathcal{A}}(q)$ is assumed to be a diagonal stable rational matrix, with diagonal elements $\mathcal{A}_j$. Introduce the controller

$$u_W(k) = (\mathbf{I} - \boldsymbol{\mathcal{W}}(q)\boldsymbol{\mathcal{R}}(q))v(k) - \boldsymbol{\mathcal{W}}(q)\boldsymbol{\mathcal{S}}(q)y(k) + \boldsymbol{\mathcal{W}}(q)\boldsymbol{\mathcal{T}}(q)r(k) \tag{2}$$

where $\boldsymbol{\mathcal{W}}$, $\boldsymbol{\mathcal{R}}$, $\boldsymbol{\mathcal{S}}$, $\boldsymbol{\mathcal{T}}$ are stable rational matrices in q, of appropriate dimension. Here, $\boldsymbol{\mathcal{W}}$ is the anti-windup filter. The controller structure proposed in (2) is inspired by a similar structure suggested in [3] for scalar systems. It is depicted in Figure 1.

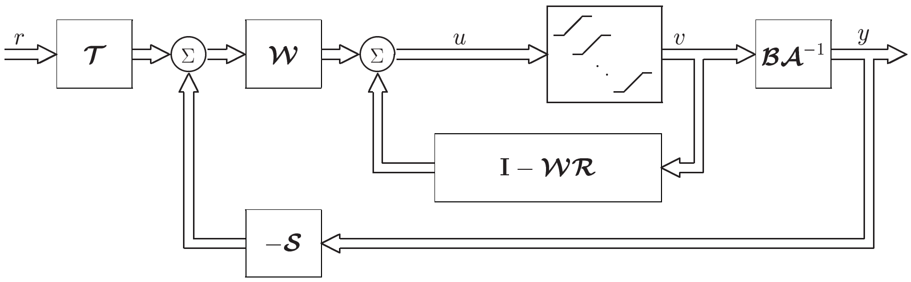


Figure 1: A discrete-time MIMO process $y(k) = \boldsymbol{\mathcal{B}}(q)\boldsymbol{\mathcal{A}}^{-1}(q)\text{sat}[u(k)]$ with a two degree of freedom controller structure $\{\boldsymbol{\mathcal{R}}(q)\ \boldsymbol{\mathcal{S}}(q)\ \boldsymbol{\mathcal{T}}(q)\}$ appended with a stable and proper anti-windup transfer-operator matrix $\boldsymbol{\mathcal{W}}(q)$. The rational matrix $\boldsymbol{\mathcal{W}}(q)$ is to be selected such that the loop gain around the saturations becomes diagonal.

Remark: In the scalar case, the controller structure (2) includes a number of well know anti-windup schemes, of which the following are worth mentioning. (The matrices $\boldsymbol{\mathcal{R}}$, $\boldsymbol{\mathcal{S}}$ and $\boldsymbol{\mathcal{T}}$ are here assumed to be scalar polynomials R, S and T.)

[1]The anti-windup concept presented here is applicable to both discrete time and continuous time systems. Here we shall however use a discrete time framework, based on the forward shift operator q, ($qy(k) = y(k+1)$).

1. *The observer-based method of Åström and Wittenmark* [7] is obtained if $\boldsymbol{\mathcal{W}} = F^{-1}$, where F is the characteristic polynomial of the anti-windup observer.
2. *The conditioning technique of Hanus* [2] is obtained if $\boldsymbol{\mathcal{W}} = t_0 T^{-1}$, where t_0 is the leading element of T.

For more details see [4, 5].

Following [4], let us regard the difference between the actual and the saturated control signal as an exogenous disturbance

$$\delta(k) \quad \triangleq \quad v(k) - u_W(k) \quad . \tag{3}$$

By omitting the argument q, and combining (1)-(3), the closed-loop system is then obtained as

$$y(k) \;=\; y_{nom}(k) + y_\delta(k) \;=\; \boldsymbol{\mathcal{H}}_{nom} r(k) + \boldsymbol{\mathcal{H}}_\delta \delta(k) \tag{4}$$

where

$$\boldsymbol{\mathcal{H}}_{nom} \;=\; \boldsymbol{\mathcal{B}}(\boldsymbol{\mathcal{R}}\boldsymbol{\mathcal{A}} + \boldsymbol{\mathcal{S}}\boldsymbol{\mathcal{B}})^{-1}\boldsymbol{\mathcal{T}} \quad ; \quad \boldsymbol{\mathcal{H}}_\delta \;=\; \boldsymbol{\mathcal{B}}(\boldsymbol{\mathcal{R}}\boldsymbol{\mathcal{A}} + \boldsymbol{\mathcal{S}}\boldsymbol{\mathcal{B}})^{-1}\boldsymbol{\mathcal{W}}^{-1} \quad . \tag{5}$$

Above $\boldsymbol{\mathcal{H}}_{nom}$ constitutes the nominal closed-loop system which is obtained in (4) when $\delta = 0$, i.e. when the control signals do not saturate. When a control signal exits saturation, $\boldsymbol{\mathcal{H}}_\delta$ will determine the resulting transient behaviour. According to the specified requirements above, the dynamics of $\boldsymbol{\mathcal{H}}_\delta$ should be fast. This can be achieved by appropriate choices of $\boldsymbol{\mathcal{W}}$. However, if the dynamics of $\boldsymbol{\mathcal{H}}_\delta$ is made too fast, then repeated saturations and limit cycles may occur. Thus, the requirements 1. and 2. above are often contradictory. It is therefore essential that an anti-windup design includes a trade-off between a fast transient and a small influence of nonlinear effects. A design method is presented next, which utilizes simple scalar tools for attaining such a trade-off .

3 Systematic anti-windup design

For the design of the anti-windup filter $\boldsymbol{\mathcal{W}}$ in (2), the criterion

$$J \;=\; \|\boldsymbol{\mathcal{H}}_\delta\|_2^2 + \left\|\mathbf{Q}\left((\boldsymbol{\mathcal{L}}_v + \mathbf{I})^{-1} - \mathbf{I}\right)\right\|_2^2 \tag{6}$$

is introduced. This criterion is a generalization of a criterion suggested for the scalar case by Sternad and Rönnbäck in [5]. In (6), $\mathbf{Q}$ is a diagonal penalty matrix whereas $\boldsymbol{\mathcal{L}}_v$ represents the loop gain around the saturation nonlinearity. Now, by choosing $\boldsymbol{\mathcal{W}}$ as

$$\boldsymbol{\mathcal{W}} \;=\; \boldsymbol{\mathcal{P}}(\boldsymbol{\mathcal{R}}\boldsymbol{\mathcal{A}} + \boldsymbol{\mathcal{S}}\boldsymbol{\mathcal{B}})^{-1} \quad , \tag{7}$$

where $\boldsymbol{\mathcal{P}}$ is a stable and rational matrix to be determined, the rational matrices $\boldsymbol{\mathcal{H}}_\delta$ and $\boldsymbol{\mathcal{L}}_v$ are given by

$$\boldsymbol{\mathcal{H}}_\delta = \boldsymbol{\mathcal{B}}\boldsymbol{\mathcal{P}}^{-1} \quad ; \quad \boldsymbol{\mathcal{L}}_v = \boldsymbol{\mathcal{P}}\boldsymbol{\mathcal{A}}^{-1} - \mathbf{I} \tag{8}$$

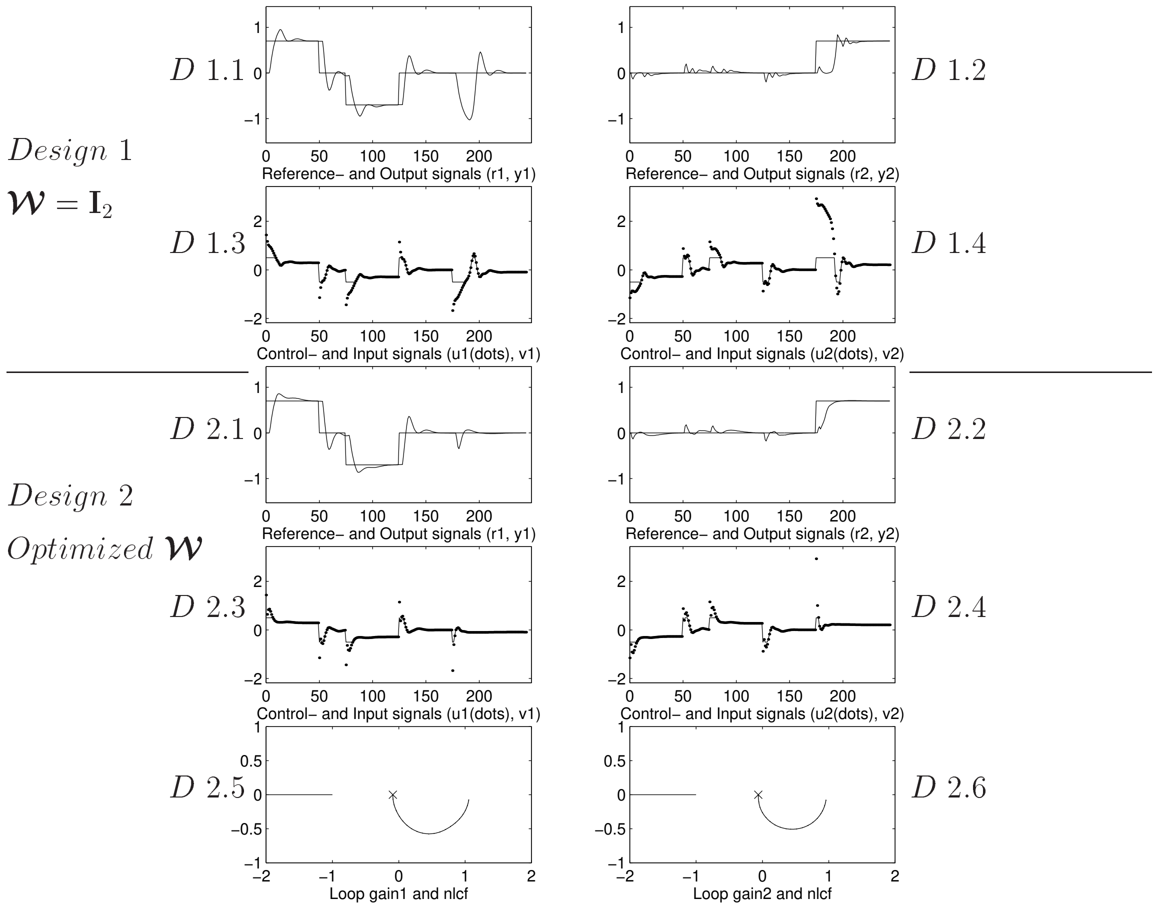
respectively. If $\boldsymbol{\mathcal{P}}$ is choosen *diagonal*, then $\boldsymbol{\mathcal{L}}_v$ will be diagonal. By insertion of (8) into (6), the criterion can be rewritten as

$$J \;=\; \left\|\boldsymbol{\mathcal{B}}\boldsymbol{\mathcal{P}}^{-1}\right\|_2^2 + \left\|\mathbf{Q}\left(\boldsymbol{\mathcal{A}}\boldsymbol{\mathcal{P}}^{-1} - \mathbf{I}\right)\right\|_2^2 \quad . \tag{9}$$

Minimizing (9), with respect to $\boldsymbol{\mathcal{P}}$, for a given penalty matrix $\mathbf{Q}$, is shown in [8] to be equivalent to the solution of m separate scalar spectral factorization equations

$$r_j \mathcal{P}_j \mathcal{P}_j^* \;=\; \sum_{i=1}^{p} \mathcal{B}_{ij}\mathcal{B}_{ij}^* + \rho_j \mathcal{A}_j \mathcal{A}_j^* \;;\; \boldsymbol{\mathcal{P}} = \mathrm{diag}(\mathcal{P}_j) \quad . \tag{10}$$

Here, (10) has to be solved for $j = 1, 2....m$, where m is the number of process inputs, p is the number of process outputs, r_j is a scale factor and ρ_j is the jth diagonal element of $\mathbf{Q}$. The design of a multivariable anti-windup compensator is thus reduced to m scalar designs, in which the m elements of the diagonal loop gain matrix $\boldsymbol{\mathcal{L}}_v$ are systematicaly adjusted. Note that if $\rho_j \to \infty$, then $\mathcal{P}_j \to \mathcal{A}_j$. The jth loop gain $\mathcal{L}_j$ will then contract and stay well away from the negative real axis. As a result, repeated saturations will not occur in that loop. However, the desaturation transients may then show an unsatisfactory behaviour, since the common denominator of the jth column of $\boldsymbol{\mathcal{H}}_\delta$ goes towards the plant dynamics $\mathcal{A}_j$. On the other hand if ρ_j is selected small, the dynamics of the jth column of $\boldsymbol{\mathcal{H}}_\delta$ will become fast, while the jth loop gain may become large. This, in turn, may generate repeated saturations and limit cycles. The user must therefore select the values of ρ_j properly to obtain an appropriate trade-off.



4 Example

The controller in (2) is used for two different choices of $\boldsymbol{\mathcal{W}}$. In both the cases, the nominal controller, $\boldsymbol{\mathcal{R}}$, $\boldsymbol{\mathcal{S}}$, $\boldsymbol{\mathcal{T}}$, originates from an observer-based state-feedback LQ-control law, expressed in input-output form. The input penalty is 0.01 for both inputs. The model used for simulation descibes a *Heavy Oil Fractionator* [1], with two inputs and two outputs. The model used for controller- and anti-windup filter design, is obtained by subspace-identification. In the first example, we select $\boldsymbol{\mathcal{W}} = \mathbf{I}$, which simply means that the observer is fed with saturated control signals. The result of the simulation is shown in the four upper diagrams, (D 1.1 – D 1.4). In the other example, the method proposed in this paper was used for the design of $\boldsymbol{\mathcal{W}}$. The result, after adjustment of the penalties ρ_1, ρ_2, is shown in the six lower diagrams, (D 2.1 – D 2.6). The two bottom diagrams show the diagonal elements of the loop gain $\mathcal{L}_v$ and the functions $-1/Y(C)$.

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