Narrowband and Broadband Multiuser Detection Using a Multivariable DFE

Claes Tidestav, Anders Ahlén and Mikael Sternad
Signal Processing Group, Uppsala University, P.O. Box 27
S–751 03 Uppsala, SWEDEN, email: ct@syscon.uu.se

ABSTRACT In this paper, we present a multivariable minimum mean square error (MMSE) DFE, which has the ability to operate on channels with different number of inputs and outputs. It is derived under the constraint of realizability and is calculated by solving a system of linear equations. The suggested DFE can be used for combined temporal and spatial equalization, as well as for narrowband, multiuser detection in systems with multiple antenna elements. The DFE may also serve as a multiuser detector in DS–CDMA systems. Simulations indicate that the DFE performs well as a multiuser detector. In a system with multiple antennas, it outperforms a corresponding detector for a single user in terms of BER while doubling the capacity. If utilized for multiuser detection in CDMA, its performance is comparable to the best achievable performance of the decorrelating detector, and is attained at a substantially lower computational cost.

1 Introduction

During the last few years, the second generation mobile cellular telephone systems have become operational. One can foresee a dramatic increase in traffic demands, both concerning transmission quality and system capacity. The transmission quality might be seriously degraded by *fading*, both Rayleigh fading and frequency selective fading. Rayleigh fading can be alleviated by means of e.g. space diversity, whereas frequency selective fading can be combatted by e.g. the *Decision Feedback Equalizer* (DFE). Only recently, combined spatial and temporal equalization has become popular, see e.g. [1]. Such algorithms jointly utilize the space diversity of the antenna system and the time diversity introduced by the channel.

Plans for the third generation of cellular systems are also being developed. The demand for high bandwidth applications has led system designers to investigate e.g. DS–CDMA. Suggested detectors for CDMA are the so-called conventional detector and the decorrelating detector.

In this paper, we propose a multivariable generalization of the DFE. The DFE is obtained by minimizing the minimum mean square error (MMSE), under the assumption of correct previous decisions. The optimization is performed *under the constraint of realizability* (finite decision delay and causal filters). This is in contrast to the multivariable DFE proposed in [2], where the author performed an *unconstrained* optimization (this corresponds to infinite decision delay in our solution). Moreover, the DFE proposed in [2] could only operate on channels with equal number of inputs and outputs. The presently proposed DFE does not have this limitation.

The proposed DFE turns out to be a very versatile algorithm. Here we will discuss three scenarios, for which the DFE could be used:

1. *Combined spatial and temporal equalization to combat Rayleigh fading as well as frequency selective fading.* The MIMO equalizer of [2] could *not* be used for this purpose, since the number of inputs differs from the number of outputs.

2. *Multiuser detection for narrowband signals using multiple antenna elements.* The multivariable DFE can be used to detect multiple users on the same channel in the same cell *simultaneously.* This problem has been studied previously by e.g. Winters [3], who proposed a method which utilized a linear equalizer. The idea to use a DFE for this scenario was suggested by Falconer *et al.* in [4]. However, the method proposed in [4] was based on diversity to enhance one message, while regarding the other messages as interference.

3. *Multiuser detection in asynchronous DS–CDMA.* Conventional detection in DS–CDMA suffer from unacceptable performance with regard to the near-far problem. Following Verdú [5], it is possible to arrive at a multivariable channel model. The most common suggestion to solve the near-far problem is the decorrelating detector. This detector, however, has been shown to be very sensitive to time delay estimation errors [6], and is in its exact form suitable only for systems employing block transmission. The use of a DFE for CDMA detection has been proposed by Duel–Hallen in [2] and by Falconer *et al.* in [4, 7]. The DFE of Duel–Hallen suffers from the limitations mentioned above, while the single variable DFE suggested by Falconer *et al.* only detects one message at a time while treating the others as interference, i.e. colored noise.

Based on the three situations above, discrete time, multivariable channel models are presented in Section 2. In Section 3, the *closed form* solution for calculating the coefficients of the MSE optimal DFE is presented. To illustrate the performance of our algorithms, Monte Carlo simulations are conducted in Section 4. Finally, conclusions are drawn in Section 5.

2 Multivariable channel models

Based on the above scenarios, we will now describe the channel models, for which the multivariable DFE is applicable. All channel models are in discrete time, obtained by sampling at the symbol rate.

2.1 Combined temporal and spatial equalization

Consider the *general* case, when there is one message and n_y receiver antennas. Let the channel from the transmitter to receiver antenna i be described by a tapped delay line

$$y_i(k) = B_i(q^{-1})d(k) + v_i(k) \ , \tag{1}$$

where q^{-1} represents the unit delay operator, $q^{-1}y(k) = y(k-1)$. In (1), $y_i(k)$ is the received signal at antenna i,

$$B_i(q^{-1}) = B_i^0 + B_i^1 q^{-1} + B_i^2 q^{-2} + \cdots + B_i^{n_b} q^{-n_b} \ ,$$

$d(k)$ is the transmitted message and $v_i(k)$ denotes the noise received at antenna i. The scenario for $n_y = 2$ is depicted in Figure 1.

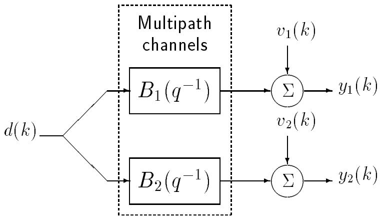

Figure 1 *Multivariable channel model resulting from diversity considerations. One message and $n_y = 2$ antennas.*

By introducing

$$y(k) = \begin{pmatrix} y_1(k) \\ \vdots \\ y_{n_y}(k) \end{pmatrix}; \quad \boldsymbol{B}(q^{-1}) = \begin{pmatrix} B_1(q^{-1}) \\ \vdots \\ B_{n_y}(q^{-1}) \end{pmatrix};$$
$$v(k) = \begin{pmatrix} v_1(k) & \ldots & v_{n_y}(k) \end{pmatrix}^T ,$$

the complete channel can be compactly written as

$$\begin{aligned} y(k) &= \boldsymbol{B}(q^{-1})d(k) + v(k) \\ &= \mathbf{B}_0 d(k) + \cdots + \mathbf{B}_{n_b} d(k-n_b) + v(k) \end{aligned} \tag{2}$$

where $\mathbf{B}_m = \begin{pmatrix} B_1^m & \ldots & B_{n_y}^m \end{pmatrix}^T$. We shall refer to (2) as *Channel Model 1.*

2.2 Narrowband multiuser detection

Consider a multiuser scenario with n_d transmitters and n_y receiver antennas. Let the channel from transmitter j to receiver antenna i be given by $B_{ij}(q^{-1})$. The received signal at base station antenna i, $y_i(k)$, can in this case be expressed as

$$y_i(k) = \sum_{j=1}^{n_d} B_{ij}(q^{-1})d_j(k) + v_i(k)$$

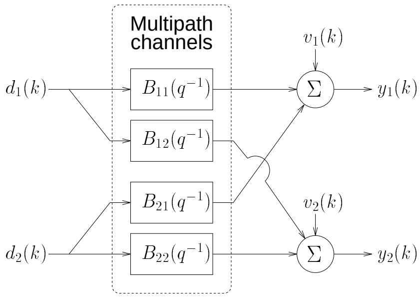

Figure 2 *Multivariable channel model resulting from a situation where multiple users share the same channel.*

By collecting the antenna signals in vector form we obtain

$$\begin{aligned} y(k) &= \begin{pmatrix} B_{11}(q^{-1}) & \ldots & B_{1n_d}(q^{-1}) \\ \vdots & \ddots & \vdots \\ B_{n_y1}(q^{-1}) & \ldots & B_{n_yn_d}(q^{-1}) \end{pmatrix} \begin{pmatrix} d_1(k) \\ \vdots \\ d_{n_d}(k) \end{pmatrix} + \begin{pmatrix} v_1(k) \\ \vdots \\ v_{n_y}(k) \end{pmatrix} \\ &= \boldsymbol{B}(q^{-1})d(k) + v(k) \\ &= \mathbf{B}_0 d(k) + \cdots + \mathbf{B}_{n_b} d(k-n_b) + v(k) \end{aligned} \tag{3}$$

where n_b is the highest degree occurring in any matrix element $B_{ij}(q^{-1})$, and

$$\mathbf{B}_m = \begin{pmatrix} B_{11}^m & \ldots & B_{1n_d}^m \\ \vdots & \ddots & \vdots \\ B_{n_y1}^m & \ldots & B_{n_yn_d}^m \end{pmatrix} .$$

The scenario is depicted in Figure 2 for $n_d = n_y = 2$. We shall in the sequel refer to (3) as *Channel Model 2.* Note that Channel Model 1 is a special case ($n_d = 1$) of Channel Model 2.

2.3 Multiuser detection in asynchronous DS–CDMA

Define $d(k) = (d_1(k)\, d_2(k) \ldots d_{n_d}(k))^T$ as the vector of transmitted symbols at time k. Following Verdú [5], the sampled output from a bank of filters, matched to the signature sequences of the individual users, may be collected in vector form

$$y'(k) = \mathbf{R}(1)\mathbf{A}d(k+1) + \mathbf{R}(0)\mathbf{A}d(k) + \mathbf{R}^T(1)\mathbf{A}d(k-1) + v(k) \ . \tag{4}$$

Here, the matrices $\mathbf{R}(m)$ contain *partial crosscorrelations* between the signature sequences used to spread the data, whereas $v(k)$ denotes noise. The diagonal matrix $\mathbf{A}$ contains channel coefficients associated with the different users. We define $y(k) = y'(k-1)$ in (4) to obtain the desired causal, multivariable channel model

$$y(k) = (\mathbf{R}(1)\mathbf{A} + \mathbf{R}(0)\mathbf{A}q^{-1} + \mathbf{R}^T(1)\mathbf{A}q^{-2})d(k) + v(k) \ . \tag{5}$$

We shall refer to (5) as *Channel Model 3.*

3 Description of the multivariable DFE

For FIR channels with white noise, the multivariable DFE has a transversal feedforward and feedback filter, see Figure 3.

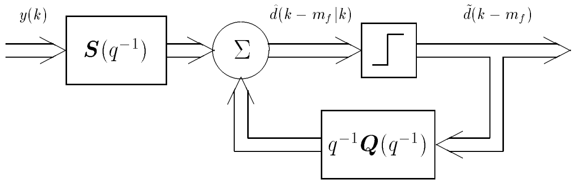

Figure 3 *Structure of the multivariable DFE*

The equalizer forms the decision variable

$$\hat{d}(k-m_f|k) = \boldsymbol{S}(q^{-1})y(k) - \boldsymbol{Q}(q^{-1})\tilde{d}(k-m_f-1) \quad (6)$$

where $\boldsymbol{S}(q^{-1})$ has n_y inputs and n_d outputs, whereas $\boldsymbol{Q}(q^{-1})$ has n_d inputs and n_d outputs. Note that the prefilter is *causal*, which means that the decision on the symbol $d(k)$ is made after a finite, user-chosen, delay m_f, the so-called *smoothing lag*. The DFE is thus always realizable!

For the determination of equalizer coefficients, we advocate an *indirect* approach. We thus suppose that a (multivariable) tapped delay line channel model has been made available to us from an identification experiment. We then compute the matrix *equalizer* coefficients from the matrix *channel* coefficients. We prefer this indirect approach to the direct approach (in which one calculates the equalizer coefficients directly from data), since it is better suited for tracking (rapidly) time-varying channels.

The MSE optimal multivariable DFE is obtained by minimizing the criterion

$$J = E[||d(k-m_f) - \hat{d}(k-m_f|k)||^2] \quad (7)$$

under the assumption of correct past decisions.

Theorem 1
Consider the DFE described in (6) and Channel Models 1 to 3 described in (2), (3) and (5), where $E[v(k)v(l)^H] = \Psi\delta_{kl}$. Assuming correct past decisions, the matrix polynomials $\boldsymbol{S}(q^{-1})$ and $\boldsymbol{Q}(q^{-1})$ of order m_f and $n_b - 1$ respectively, minimizing (7), are obtained as follows:

1. Solve the system of linear equations

$$\left(\begin{array}{ccc|ccc} \mathbf{B}_0^H & \dots & \mathbf{0} & \mathbf{I}_{n_d} & \dots & \mathbf{0} \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ \mathbf{B}_{m_f}^H & \dots & \mathbf{B}_0^H & \mathbf{0} & \dots & \mathbf{I}_{n_d} \\ \hline -\Psi & \dots & \mathbf{0} & \mathbf{B}_0 & \dots & \mathbf{B}_{m_f} \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ \mathbf{0} & \dots & -\Psi & \mathbf{0} & \dots & \mathbf{B}_0 \end{array}\right) \left(\begin{array}{c} \mathbf{S}_0^H \\ \vdots \\ \mathbf{S}_{m_f}^H \\ \hline \mathbf{L}_{1m_f} \\ \vdots \\ \mathbf{L}_{10} \end{array}\right)$$
$$= \left(\begin{array}{cccc|ccc} \mathbf{0} & \dots & \mathbf{0} & \mathbf{I}_{n_d} & \mathbf{0} & \dots & \mathbf{0} \end{array}\right)^T \quad (8)$$

with respect to the matrices $\boldsymbol{S}_i^H$ and $\boldsymbol{L}_{1j}$. The feedforward filter is then given by $\boldsymbol{S}(q^{-1}) = \mathbf{S}_0 + \mathbf{S}_1 q^{-1} + \dots + \mathbf{S}_{m_f} q^{-m_f}$.

2. Calculate the coefficients of the feedback filter $\boldsymbol{Q}(q^{-1}) = \mathbf{Q}_0 + \mathbf{Q}_1 q^{-1} + \dots + \mathbf{Q}_{n_b-1} q^{-n_b+1}$ according to

$$\boldsymbol{Q}_i = \sum_{j=0}^{m_f} \boldsymbol{S}_{m_f-j} B_{j+i+1} \quad (9)$$

■

Proof: See [8].

Remark. Note that the algorithm presented in Theorem 1 requires *no spectral factorization.*

4 Monte Carlo simulations

To explore the performance of the multivariable DFE, several Monte Carlo simulations were conducted. The primary goal was to test the multivariable DFE as an algorithm for multiuser detection in two applications: multiuser detection in multiple antenna systems and multiuser detection in DS–CDMA systems. The following properties of the multivariable DFE were investigated.

(a) For Channel Model 2
 - the performance when the channels are exactly known
 - the performance when the channels are identified.

(b) For Channel Model 3
 - the performance for exactly known channels and exactly known propagation delays.

In all simulations, the channels were subject to Rayleigh fading, with data transmitted in bursts. Over a burst, all channels were *time-invariant.* The modulation scheme was BPSK, and the smoothing lag m_f was chosen to be equal to the length of the channel impulse response.

4.1 Multiuser detection using multiple antenna elements

4.1.1 Known channels

In this simulation, which represents an uplink scenario[1], we have two mobiles and a base station with two antenna elements. Each channel has three taps of equal average power, subject to independent Rayleigh fading. Also, the channels from one mobile to different base station antenna elements are assumed to be independent. The same assumption is made for the channels from different mobiles to the same base station antenna element. The assumption of uncorrelated antenna elements at the base station is idealized. In practice the correlation will depend on e.g. the distance between the antennas and the conditions of the antenna site. For an elaborate discussion of this subject, see [8].

The noise consists of two co-channel interferers, plus additional white noise; the "interference-to-noise ratio" is 20 dB. The co-channel interferers have the same character

[1] Downlink scenarios could be considered as well by assuming multiple antennas at the mobile.

as the mobiles. The only difference is that their channels have two taps instead of three, which is a reasonable assumption since interferers are generally more remote.

The above scenario is simulated for average signal-to-interference-plus-noise (SINR:s) from 5 dB to 20 dB, where SINR is defined as

$$\mathrm{SINR}_{ij} = \frac{E[|B^0_{ij}|^2 + |B^1_{ij}|^2 + |B^2_{ij}|^2]E[|d(k)|^2]}{E[|v_i(k)|^2]}$$

In one case, the two transmitters are assumed to be of equal power (i.e. $\mathrm{SINR}_{i1} = \mathrm{SINR}_{i2}$), and in the other case, one transmitter is 20 dB stronger than the other one (i.e. $\mathrm{SINR}_{i2} = \mathrm{SINR}_{i1}$+20 dB). As a comparison, a single-user-single-antenna case is included. This case is obtained by removing one mobile and one antenna. All other conditions remain the same. Note that the signal from the mobile in the same cell is *not* considered as interference, in contrast to the approach of Falconer *et al.* in [4]. A total number of 10 000 randomly chosen channels were simulated. Over each channel, a burst containing 100 symbols was transmitted.

Figure 4 shows the *average* BER of the two detected sequences as a function of SINR ($\mathrm{SINR} = \mathrm{SINR}_{11} = \mathrm{SINR}_{21}$). As can be seen from Figure 4, the average BER for the two transmitters was slightly higher when one source was stronger than the other. When the transmitters were equally strong, their *individual* BER:s were, quite naturally, identical. On the other hand, when the transmitter powers differed by 20 dB, all the errors encountered during the transmission were found in the message of the weaker transmitter. However, in both cases the average BER was lower than the BER for the single-user-single-antenna case.

4.1.2 Identified channels

To demonstrate how the multivariable DFE works in a more realistic case, channel estimation is introduced. The data is transmitted in bursts, with a structure similar to

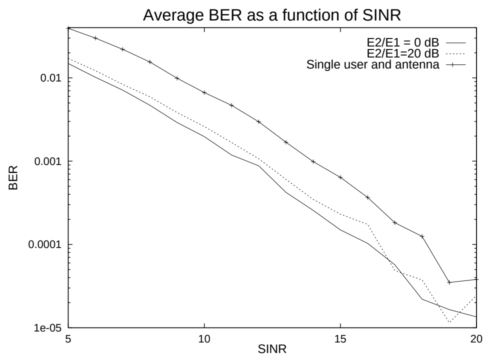

Figure 4 *The performance of the multivariable DFE with exactly known channels. Two receiver antennas and two simultaneous messages. Single-user-single-antenna case (solid with stars), two-user case, $E_2/E_1 = 0$ dB (solid) and $E_2/E_1 =$ 20 dB (dotted)*

that of GSM: a training sequence of 26 symbols is located in the middle of each burst. Together with data bits, tail bits and control bits, this results in a total burst length of 148 bits. The channel estimation is performed using the off-line least squares method. Apart from this, the simulation conditions are the same as in subsection 4.1.1. The results are indicated in Figure 5.

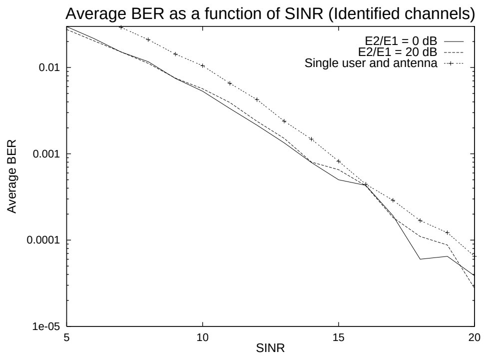

Figure 5 *The performance of the multivariable DFE when the channel has been identified in a time slot structure similar to that of GSM. Single-user-single-antenna case (dashed with stars), two-user case, $E_2/E_1 = 0$ dB (solid) and $E_2/E_1 =$ 20 dB (dashed)*

The important features obtained from the simulations with known channels were preserved when channel estimation was introduced: the average BER was somewhat higher when the transmitter powers of the mobiles differed, and the single-user-single-antenna case was still inferior to the two-user case. The difference is however smaller, since the channel estimation is harder in a two-user case: the number of parameters to estimate is quadrupled, whereas the number of available data is only doubled.

4.2 Multiuser detection in asynchronous DS–CDMA

The simulation is conducted based on a two-user scenario with spreading performed using Gold sequences of length 31. The transmitter powers differ by 20 dB to simulate a near-far problem scenario. Each channel is assumed to be accurately described by an exactly known, one-tap impulse response, subject to Rayleigh fading. The exactly known time delays between the users are uniformly distributed over a symbol period. The experiment involved a simulation of 1000 channels and for each channel 100 data symbols were transmitted.

The symbols were detected using the conventional detector, the decorrelating detector and the multivariable DFE. Figure 6 shows the BER:s obtained from the simulation.

The conventional detector is clearly inferior to the other two algorithms, since the it is not near-far resistant. The signal energy from the stronger transmitter will leak into the detector for the weaker one, causing high BER:s. The decorrelating detector and the multivariable DFE perform equally well.

The decorrelating detector is implemented in its exact form suitable *only* for block transmission. The detection involves the inversion of a matrix of dimension (number of transmitters) $\times$ (number of symbols in a burst). Recently, results have been published [6] which show that the decorrelating detector is sensitive to errors in the propagation delay estimation.

The filter structure of the DFE enables operation even when continuous transmission is employed. To obtain a decorrelating detector with filter structure, approximation would be necessary, so performance degradation could be expected. One could also expect the multivariable DFE to be superior for channel models with more than one tap.

Because of its good performance and reasonable complexity, the multivariable DFE seems to be a good candidate for multiuser detection in asynchronous DS–CDMA. When it comes to robustness against propagation delay estimation errors as well as to channel coefficient estimation errors, the DFE must, of course, be thoroughly examined. The results in subsection 4.1.2 indicate that the DFE works well even when the channel coefficients are not exactly known.

5 Conclusion

In this paper, we have suggested the use of a multivariable DFE. This DFE can be utilized for several scenarios. First, it can be used for *combined temporal and spatial equalization*. Second, the multivariable DFE can be used as a tool for narrowband multiuser detection in a situation with multiple antenna elements at the base station or at the mobile. The detector for this scenario provides a lower (average) bit error rate than a corresponding single-user-single-antenna scenario. We have shown that this result holds for a GSM-like system even when channel estimation is introduced. Finally, it is possible to use the equalizer for multiuser detection in asynchronous DS–CDMA. In a near-far situation, the BER of the DFE is lower than for the conventional detector and comparable to the BER of the decorrelating detector. The decorrelating detector is however more complex and can only be used in systems employing block transmission. It is also sensitive to propagation delay estimation errors. Robustness with respect to such errors has not been thoroughly investigated for the DFE, but the results in subsection 4.1.2 indicate that the DFE would work well also in a case where the channel is not exactly known. Hence, the multivariable DFE could be expected to perform better than the decorrelating detector in a realistic case.

The computational complexity of the DFE algorithm is clearly reasonable. The calculation of the equalizer requires only the solution of a system of linear equations.

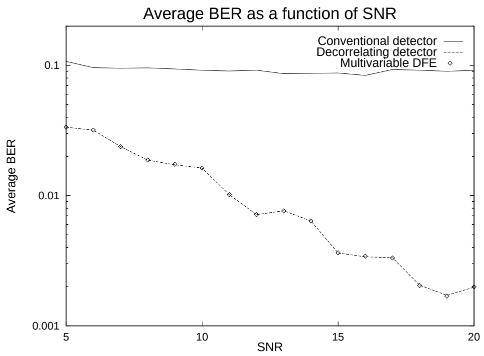

Figure 6 *The multivariable DFE used as a CDMA detector (diamonds), compared to the conventional detector (solid) and the decorrelating detector (dashed).*

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