In Proceedings of Nordic Radio Symposium seminar, Uppsala, Sweden, November 10-11, 1993.

DESIGNING DECISION FEEDBACK EQUALIZERS TO BE ROBUST WITH RESPECT TO CHANNEL TIME VARIATIONS

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Abstract: Design equations are presented for a robust realizable decision feedback equalizer, for FIR channels with uncertain channel coefficients and white noise. The mean MSE averaged over the class of channels is minimized. An example using a robust DFE for a fading GSM-channel is presented.

1. INTRODUCTION

If data sequences $\{d(n)\}$ are transmitted in the presence of intersymbol interference, they have to be reconstructed from the received sequences $\{y(n)\}$. Equalizers compute estimates $\bar{d}(n)$ on a symbol by symbol basis. Their main advantage, as compared to the MLSE Viterbi detector, is a low computational complexity.

Figure 1: Decision feedback equalizer

Due to reflections from surrounding objects, the reception for a mobile radio varies as its location changes. This causes time variations (fading) in the coefficients of the channel.

If the time variations are large an equalizer that adapts to the channel has to be used. If the time variations are small, the filter coefficients can be adjusted during known training sequences, and held fixed until the next training. However if in the latter case the time variations are not negligible, they have to be considered when designing the equalizer.

A method for dealing with this problem will be presented in Section 3 below. It utilizes a technique for taking the spectral uncertainity of IIR models into account in the design of Wiener filters, which has recently been presented in [6],[8]. In this approach, a set of possible systems around the nominal design model is parametrized by random variables. That way of describing model uncertainity is closely related to the stochastic embedding concept developed by Goodwin and co-writers in [3],[4]. A robust (realizable) Wiener filter is then obtained by minimizing the MSE, *averaged* over the set of possible true systems. The design equations become no more complicated than for a Wiener filter which does not take model uncertainity into account.

To make a robust filter sufficiently insensitive, but not too cautious, the set of possible systems must be covered by the model error description. In Section 4 below, we will illustrate how this can be acheived in a mobile radio application.

2. MODEL AND FILTER STRUCTURE

Consider the following received, discrete-time, complex baseband signal

$$y(n) = \left(B_0(q^{-1}) + \Delta B(q^{-1})\right) d(n) + v(n) \qquad (1)$$

where q^{-1} is the delay operator such that $q^{-1}y(n) = y(n-1)$). The transmitted symbols $\{d(n)\}$ are assumed to be zero mean and white, with $E|d(n)|^2 = \sigma_d^2$. The noise $v(n)$ is zero mean, with variance $E|v(n)|^2 = \sigma_v^2$.

The nominal model of the transmission channel is described by a FIR filter with complex coefficients

$$B_0(q^{-1}) = b_0 + b_1 q^{-1} + \ldots + b_n b q^{-nb} \qquad (2)$$

The set of possible model errors is represented by the "error model"

$$\Delta B(q^{-1}) = \Delta b_0 + \Delta b_1 q^{-1} + \ldots + \Delta b_{nb} q^{-nb} \qquad (3)$$

The coefficients Δb_i are stochastic variables with zero mean and known correlations $E[\Delta b_i \Delta b_j^*]$. This channel model is a special case of more general uncertain models discussed in [6] and [8].

We now introduce the following IIR decision feedback equalizer (DFE)

$$\hat{d}(n-\ell|n) = \frac{S(q^{-1})}{R(q^{-1})} y(n) - \frac{Q(q^{-1})}{P(q^{-1})} \bar{d}(n-\ell-1) \quad (4)$$

where ℓ is a user-chosen smoothing lag and $\bar{d}(n)$ are decisioned data. The denominator polynomials

$R(q^{-1})$ and $P(q^{-1})$ are required to be monic and stable. The optimization of such equalizers based on exactly known channel and noise models has been discussed in e.g.[5].

Figure 2: Channel with error modell and IIR decision feedback equalizer.

In the theoretical treatment of the equalizer, decided data $\bar{d}(n-\ell-1)$ is replaced by delayed transmitted data. By doing this, we eliminate the nonlinear decision device from the theoretical treatment.

3. FILTER DESIGN EQUATIONS

We have designed an equalizer which is optimal in the sense that it minimizes the *averaged MSE criterion*

$$J = \bar{E}E|d(n-\ell) - \hat{d}(n-\ell|n)|^2 \qquad (5)$$

where E represents expectation over d and v and $\bar{E}$ is an expectation over the model error distribution in (1). This type of criterion has been used in connection to other filtering problems, e.g. by Chung and Bélanger [2]. Note that not only the *range* of uncertainties, but also their *likelihood* is taken into account by (5); common model deviations will have a greater impact on an estimator design than do very rare "worst cases".

In [7], design equations have been derived for minimizing the criterion (5) with respect to the filter coefficients of the DFE (4), for an ensemble of systems (1). A novel derivation technique, described in [1] and in [6], has been used.

Figure 3: Theoretical model of the decision feedback equalizer.

For polynomials

$$P = p_0 + p_1 q^{-1} + \dots \qquad (6)$$

let

$$P_* \triangleq p_0^* + p_1^* q + \dots \qquad (7)$$

where q is the forward shift operator.
Define

$$\rho = \sigma_v^2/\sigma_d^2 \quad .$$

The polynomials $S(q^{-1})$, $R(q^{-1})$, $Q(q^{-1})$ and $P(q^{-1})$ in the equalizer (4) can then be calculated as follows.

Let the scalar r_1 and the stable monic polynomial $\beta(q^{-1})$ be the solution to the *averaged spectral factorization*

$$r_1 \beta\beta_* = \bar{E}(\Delta B \Delta B_*) + \rho \qquad (8)$$

Let $\{Q(q^{-1}), S(q^{-1}), L_{1*}(q), L_{2*}(q)\}$ be the (unique) solution to the *coupled polynomial equations*

$$\beta + q^{-1}Q = q^{\ell} B_0 S + \beta L_{1*} \qquad (9)$$

$$qL_{2*} = -r_1 \beta_* S + q^{-\ell} B_{0*} L_{1*} \qquad (10)$$

The polynomial degrees are

$$\deg S = \deg L_1 = \ell$$

$$\deg Q = \deg L_2 = \max\{\deg B_0, \deg \beta\} - 1 \quad .$$

Then, the equalizer (4), which minimizes (5), is given by Q and S from (9) and (10) and by

$$R = \beta \quad ; \quad P = \beta \quad . \qquad (11)$$

This result can be derived by adding a variation $\nu(n) = \mathcal{T}_1 y(n) + \mathcal{T}_2 \bar{d}(n-\ell-1)$, with $\mathcal{T}_1$ and $\mathcal{T}_2$ arbitrary but rational and stable, to the estimate $\hat{d}(n-\ell|n)$. Optimality corresponds to orthogonality of the mean estimation error with respect to this variation. The equations (9) and (10) arise from requiring orthogonality with respect to the two terms $\mathcal{T}_1 y$ and $\mathcal{T}_2 \bar{d}$, separately. A detailed derivation can be found in [7].

The coupled equations (9),(10) can be solved by converting them to a system of linear equations in the coefficients.

Define

$$\boldsymbol{B} \triangleq \begin{pmatrix} b_0 & 0 & \dots & 0 \\ b_1 & b_0 & \dots & 0 \\ \dots & \dots & \dots & \dots \\ b_\ell & b_{\ell-1} & \dots & b_0 \end{pmatrix} \qquad (12)$$

$$\boldsymbol{\beta} \triangleq \begin{pmatrix} 1 & 0 & \ldots & 0 \\ \beta_1 & 1 & \ldots & 0 \\ \ldots & \ldots & \ldots & \ldots \\ \beta_\ell & \beta_{\ell-1} & \ldots & \beta_0 \end{pmatrix} \quad (13)$$

$$\boldsymbol{S}^T \triangleq \begin{pmatrix} s_0 & s_1 & \ldots & s_\ell & l^*_{1,\ell} & l^*_{1,\ell-1} & \ldots & l^*_{1,0} \end{pmatrix} \quad (14)$$

$$\boldsymbol{C}^T \triangleq \begin{pmatrix} 0 & 0 & \ldots & 1 & 0 & 0 & \ldots & 0 \end{pmatrix} \quad (15)$$

where the "1" in $\mathbf{C}$ appears in element nr $\ell+1$. Then it can be shown from (9) and (10) that

$$\boldsymbol{S} = \begin{pmatrix} \boldsymbol{B} & \boldsymbol{\beta} \\ r_1 * \boldsymbol{\beta}^* & -\boldsymbol{B}^* \end{pmatrix}^{-1} \boldsymbol{C} \quad (16)$$

From $\boldsymbol{S}$ we now can extract S (and L_1) and subsequently Q can be obtained from equation (9).

Solution of the spectral factorization (8) is straightforward. With a given right–hand side, it is just an ordinary polynomial (FIR) spectral factorization, for which there exist efficient iterative algorithms.

The averaged term, $\bar{E}(\Delta B \Delta B_*)$, in(8) can be evaluated as follows. For a stochastic polynomial $\Delta B(q^{-1})$, of degree nb, let the Hermitian parameter covariance matrix be

$$\mathbf{P}_{\Delta B} = \begin{bmatrix} \bar{E}|\Delta b_0|^2 & \ldots & \bar{E}(\Delta b_0 \Delta b^*_{nb}) \\ \vdots & \ddots & \vdots \\ \bar{E}(\Delta b_{nb} \Delta b^*_0) & \ldots & \bar{E}|\Delta b_{nb}|^2 \end{bmatrix}$$

Denote the sum of the diagonal elements g_0, the sum of the elements in the i'th super–diagonal g_i, the sum of elements in the i'th subdiagonal g_{-i}. Note that $g_{-i} = g^*_i$. Then it becomes evident, by direct multiplication of $\Delta B(q^{-1})\Delta B_*(q)$, and taking expectations, that

$$\bar{E}(\Delta B \Delta B_*) = g^*_{db} q^{-db} + \ldots + g^*_1 q^{-1} + g_0 + g_1 q + \ldots + g_{db} q^{db}$$

Above, $db \leq nb$, with $db = 0$ for uncorrelated coeff.

Note that, apart from the nominal model and the variance ratio ρ, only second order moments of the model error distributions need to be known.

If the coefficients in ΔB are uncorrelated, then $\bar{E}(\Delta B \Delta B_*)$ is a constant (not a polynomial) and from equation (8) we see that the robust equalizer is achieved by adding the sum of the variances of the coefficients of ΔB to the variance ratio ρ. Thus, if the uncertainities of the channel coefficients are uncorrelated, the robust equalizer is obtained by increasing the noise power for which the equalizer is designed.

Figure 4: Example of the variation of a channel tap at carrier frequency 1800 MHz and mobile speed 200 km/h. The time span is 58 symbols of 3.7μs each. Solid line: real part, dashed line: imaginary part.

4. EXAMPLE

Consider as an example a GSM-channel with three independently Rayleigh fading received signals having average relative powers 0 dB, -1.8 dB and -4.8 dB. The three rays are arriving separated by one symbol interval. The carrier frequency used is 1800 MHz. The GSM-system uses a partial response modulation stretching over 3-4 symbol intervals. This results in a channel with 5-6 coefficients b_i.

Fading due to movement of the mobile causes the channel to be slightly time varying over the duration of a burst. An example of this time variation can be seen in Fig. 4.

This time variation can be taken into account when designing a robust DFE. The channel identified during the training sequence is used for the nominal model $B_0(q^{-1})$. The time variation is viewed as one part of the stochastic uncertainity $\Delta B(q^{-1})$. Another part of the $\Delta B(q^{-1})$ is the uncertainity in the identification of the channel during the training sequence.

In order to design the equalizer, an average value for the correlation matrix $P_{\Delta B}$ is required. $P_{\Delta B}$ consists of two parts

$$P_{\Delta B} = P_{\text{Time variation}} + P_{\text{Identification uncertainity}} \quad (17)$$

The correlation matrix for the channel coefficients of the channel P_B can be estimated on line by averaging over a number of bursts. $P_{\text{Time variation}}$ can

Figure 5: Performance gain, BER_{nom}/BER_{rob} and MSE_{nom}/MSE_{rob}, for the robust equalizer as compared to the nominal equalizer.

Figure 6: BER for the robust and the nominal DFE:s.

then be related to P_B by

$$P_{\text{Time variation}} = 2P_B(1 - \frac{1}{T}\int_0^T J_0(2\pi f_c \frac{v_0}{c_0}\tau)d\tau) \tag{18}$$

where T is the time period averaged over, f_c is the carrier frequency, c_0 is the speed of light and v_0 is the speed of the mobile, which can be estimated.

$P_{\text{Identification uncertainity}}$ can be computed, by well known methods, from the training sequence and the estimated noise variance. With the resulting, $P_{\Delta B}$, the robust equalizer can now be designed.

The performance gained with the robust DFE as compared to the nominal DFE can be seen in Fig. 5. The performance gain increases with increasing speed of the mobile, and is largest at large SNR's.

In the considered example, it turns out that the uncertainities in the coefficients of the channel are only weakly correlated and therefore one could design an almost as good robust equalizer by only adding the sum of the diagonal of $P_{\Delta B}$ to the variance ratio ρ.

References

[1] A. Ahlén and M. Sternad, "Wiener filter design using polynomial equations". *IEEE Trans. Signal Processing*, vol 39, pp 2387–2399, 1991.

[2] R. C. Chung and P. R. Bélanger, "Minimum–sensitivity filter for linear time–invariant stochastic systems with uncertain parameters". *IEEE Trans. Aut. Control*, vol 21, pp 98–100, 1976.

[3] G. C. Goodwin and M. E. Salgado, "A stochastic embedding approach for quantifying uncertainty in the estimation of restricted complexity models". *International Journal of Adaptive Control and Signal Processing*, vol 3, pp 333–356, 1989.

[4] G. C. Goodwin, M. Gevers and B. Ninnes, "Quantifying the error in estimated transfer functions with application to model order selection". *IEEE Trans. Aut. Control*, vol 37, pp. 913–928, 1992.

[5] M. Sternad and A. Ahlén, "The structure and design of realizable decision feedback equalizers for IIR channels with colored noise". *IEEE Trans. Information Theory*, vol 36, pp 848–858, 1990.

[6] M. Sternad and A. Ahlén, "Robust filtering and feedforward control based on probabilistic descriptions of model errors". *Automatica*, vol 29, pp 661-679, 1993.

[7] M. Sternad, A. Ahlén and E. Lindskog, "Robust decision feedback equalization". In preparation.

[8] M. Sternad, A. Ahlén and E. Lindskog, "Robust decision feedback equalizers". ICASSP 1993, Minneapolis, MN, USA, April 27-30 1993, vol III, pp 555-558.