Adaptive Modulation Systems for Predicted Wireless Channels

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***Abstract*— When adaptive modulation is used to counter short-term fading in mobile radio channels, signaling delays create problems with outdated channel state information. The use of channel power prediction will improve the performance of the link adaptation. It is then of interest to take the quality of these predictions into account explicitly when designing an adaptive modulation scheme. We study the optimum design of an adaptive modulation scheme based on uncoded M-QAM modulation assisted by channel prediction for the flat Rayleigh fading channel. The data rate, and in some variants the transmit power, are adapted to maximize the spectral efficiency subject to average power and bit error rate constraints. The key issues studied here are how a known prediction error variance will affect the optimized transmission properties such as the SNR boundaries that determine when to apply different modulation rates, and to what extent it affects the spectral efficiency. This investigation is performed by analytical optimization of the link adaptation, using the statistical properties of a particular but efficient channel power predictor. Optimum solutions for the rate and transmit power are derived based on the predicted SNR and the prediction error variance.**

I. Introduction

Considering the rapidly growing demand for mobile communications while limited spectrum is available, spectrally efficient communication techniques are of great importance in future wireless communications. Adaptive modulation, or link adaptation, is a powerful technique for improving the spectral efficiency in wireless transmission over fading channels and has been extensively studied in [1]–[6] and the references therein. With the adaptive modulation considered here, a high spectral efficiency is attainable at a given Bit Error Rate (BER) in favorable channel conditions, while a reduction of the throughput is experienced when the channel degrades. The adaptation can also take requirements of different traffic classes and services such as required BERs, into account.

We consider *fast* link adaptation, i.e. we strive to adapt to the small scale fading. The receiver estimates the received power and sends feedback information via a return channel to the transmitter, with the aim of modifying the modulation parameters. Due to the unavoidable delays involved in power estimation, feedback transmission and modulation adjustment, the fast link adaptation needs to be based on *predicted* estimates of the power of the fading communication channel. In the so far proposed solutions for optimum design of adaptive modulation systems, perfect knowledge of the CSI at the transmitter as well as error free channel estimates at the receiver are common assumptions for the system design and performance evaluation. In real systems, these assumptions are not valid. Due to the time-varying nature of the wireless channels, the channel status will change during the time delay between estimation and data transmission.

The impact of the uncertainty in channel estimates on the performance has been discussed in the literature (see e.g. [1], [2], [7]–[9]). In [1], [2], the impact of time delay on the adaptive modulation performance is characterized. It is shown that systems with low BER requirements are very sensitive to the time delay. In [8], a linear predictor is used to estimate the current channel status based on the outdated estimates. The channel status is used to determine the currently appropriate modulation. However, the SNR thresholds which determine the modulation modes are evaluated based on simulation results only. Results by Goeckel [7] highlight that time variations of the channel should be taken into account. In [9], adaptive modulation schemes based on very accurate long-rang CSI prediction are investigated.

The system proposed here utilizes an unbiased quadratic regression of historic noisy channel estimates to predict the signal power at the receiver [10], [11]. For this type of predictor, there exists a statistical model for the prediction error for Rayleigh fading channels which enables an analytical optimization of the rate adaptation scheme. This statistical model will also be used for analyzing the resulting BER and spectral efficiency for given prediction error variances. We restrict our attention to link adaptation with uncoded M-QAM modulation. With no coding, the two remaining degrees of freedom are the choice of modulation formats in different SNR regions, and the possibility to use transmit power control within these regions. Exploitation of the statistical information about the prediction errors will be shown to improve the overall system performance, and adjust the link adaptation better to the true channel conditions. As a result, the BER constraints will be fulfilled also in the presence of prediction errors. This will not be the case if the prediction errors are neglected in the link adaptation design.

This paper is organized as follows. Section II describes the system model and the notations which are used throughout this study. The channel prediction is explained in Section III. The BER is evaluated as a function of predicted instantaneous SNR in Section IV and optimal rate and power adaptation schemes are derived under different constraints in Section V. Analytical results are presented in Section VI while Section VII summarizes the results.

II. System Model

In the adaptive modulation scheme, M-QAM modulation schemes with different constellation sizes are provided at the transmitter. The channel is modelled by a flat Rayleigh fading channel. At the receiver, demodulation is performed using channel estimates. Fig. 1 shows the discrete model of the system. All the signals are sampled at the symbol rate where the index n represents the signal sample at time nT_s where T_s is the symbol period. Here, g_n is the zero mean, complex channel gain with circular Gaussian distribution where the power $|g_n|^2$ is $\chi^2(2)$ distributed. The auto-

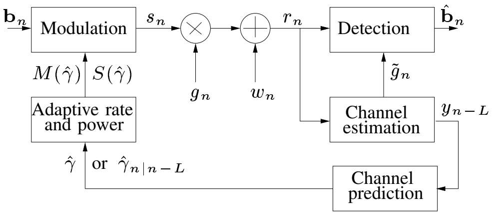

Fig. 1. Discrete model of the system.

correlation function of the complex channel gain is denoted by

$$r_g(m) = E(g_n g_{n-m}^*). \quad (1)$$

In the following, r_g will denote the average channel power gain $E|g_n|^2 = r_g(0)$. Moreover, w_n is a sample of a complex Additive White Gaussian Noise (AWGN) with zero mean and time-invariant variance σ_w^2. The estimate y_{n-L} is the noisy observation of g_{n-L} at the receiver. A time-series of these estimates are used at the receiver to predict the channel power gain $|g_n|^2$ which is proportional to the instantaneous received SNR, denoted by γ_n. Either the predicted SNR, denoted by $\hat{\gamma}_{n|n-L}$ or $\hat{\gamma}$, or the corresponding appropriate rate and transmission power levels are then fed back to the transmitter. An error free feed-back channel is assumed. The prediction horizon L is assumed to be equal the sum of computational delays and signalling delays in the adaptation control loop. Based on $\hat{\gamma}$, a modulation scheme with constellation size $M(\hat{\gamma})$ (out of N constellations available at the transmitter), with $k(\hat{\gamma}) = \log_2 M(\hat{\gamma})$ bits per symbol, and a transmit power $S(\hat{\gamma})$ are selected. Each block of $k(\hat{\gamma})$ data bits denoted by $\mathbf{b}_n$, is Gray encoded and mapped to a symbol in the signal constellation denoted by s_n, which is transmitted over the flat Rayleigh fading channel. The received samples, $\{r_n\}$, provide an estimate of the channel gain $\tilde{g}_n$, which in turn is used to obtain the detected block $\hat{\mathbf{b}}_n$. Since the estimation error in $\tilde{g}_n$ is believed to have a minor effect on the performance compared to the prediction error, perfect channel estimation is here assumed for the demodulation. In this study, the following notations similar to those of [6] are used. Let $\bar{S}$ denote the average transmit signal power. The average received SNR is then given by $\bar{\gamma} = r_g \frac{\bar{S}}{\sigma_w^2}$. For a constant transmit power $\bar{S}$, the instantaneous received SNR is $\gamma_n = \bar{\gamma}\frac{p_n}{r_g}$, where $p_n = |g_n|^2$ is the instantaneous channel power gain. The predicted received SNR is

$$\hat{\gamma} = \hat{\gamma}_{n|n-L} = \bar{\gamma}\frac{\hat{p}_{n|n-L}}{r_g} \quad (2)$$

where $\hat{p}_{n|n-L}$ is the predicted instantaneous channel power gain $|g_n|^2$. For the transmit power $S(\hat{\gamma})$, the instantaneous received SNR is given by $\gamma_n(S(\hat{\gamma})/\bar{S})$. The rate region boundaries, defined as the ranges of $\hat{\gamma}$ values over which the different constellations are used by the transmitter, are denoted by $\{\hat{\gamma}_i\}_{i=0}^{N-1}$. When the predicted instantaneous SNR belongs to a given rate region, i.e. $\hat{\gamma} \in [\hat{\gamma}_i, \hat{\gamma}_{i+1})$, the corresponding constellation of size $M(\hat{\gamma}) = M_i$ with $k(\hat{\gamma}) = k_i$ bits per symbol is transmitted where $\hat{\gamma}_N = \infty$. There is no transmission if $\hat{\gamma} < \hat{\gamma}_0$, i.e. $\hat{\gamma}_0$ is the cutoff SNR.

III. Channel Prediction

The absolute square, i.e. the power, of the time series g_n is to be predicted based on observations y_n that are assumed to be affected by an additive estimation error e_n. Thus, $y_n = g_n + e_n$. Here e_n is assumed to be a white and zero mean complex Gaussian random variable which is independent of g_n. Based on a finite number of past observations of y_n, the complex channel at time n could be predicted with a prediction horizon L by a linear FIR filter

$$\hat{g}_{n|n-L} = \varphi_{n-L}^H \theta \quad (3)$$

where θ is a column vector containing K complex-valued predictor coefficients and

$$\varphi_{n-L}^H = [y_{n-L}, y_{n-L-1}, \cdots, y_{n-L-(K-1)}], \quad (4)$$

is the regressor vector where H represents a Hermitian transpose. A Wiener adjustment of θ provides the optimal linear predictor in the Mean Square Error (MSE) sense.

The adjustment of an adaptive modulation scheme is determined not by the complex channel gain g_n, but by the SNR at the time of transmission. If we, for simplicity, assume that the variance of the noise w_n in Fig. 1 is constant, the channel power $p_n = |g_n|^2$ will have to be predicted. However, the use of the squared magnitude of the linear prediction $\hat{g}_{n|n-L}$ as a predictor of the channel power would on average underestimate the true power, and result in a biased estimate. The reason is that the average power of $\hat{g}_{n|n-L}$ will decrease with an increasing prediction horizon L and be lower than the average power of g_n, due to the limited predictability of the process g_n. We here instead utilize a recently developed quadratic power predictor which eliminates this bias. It is given by

$$\hat{p}_{n|n-L} = \theta^H \varphi_{n-L}\varphi_{n-L}^H \theta + r_g - \theta^H \mathbf{R}_\varphi \theta. \quad (5)$$

Here, $\mathbf{R}_\varphi = E(\varphi_{n-L}\varphi_{n-L}^H)$ is the $K \times K$ correlation matrix for the regressors. Note that $E(\hat{p}_{n|n-L}) = r_g$ for all L. The unbiased quadratic predictor that minimizes the power MSE is derived in [10], where it is shown that the predictor coefficient vector θ that provides an MSE optimal the channel predictor (3) will *also* result in an MSE optimal power predictor, when used in (5). The optimal adjustment for both of these problems is thus given by

$$\theta = \mathbf{R}_\varphi^{-1}\mathbf{r}_{g\varphi}, \quad (6)$$

$$\mathbf{r}_{g\varphi} = [r_g(L), r_g(L+1), \cdots, r_g(L+(K-1))]^T. \quad (7)$$

Assume that the second order statistics of g_n have been estimated perfectly, so that the parameter vector θ is perfectly adjusted. Then, the miminum MSEs of the channel gain prediction error $\epsilon_{c_n} = g_n - \hat{g}_{n|n-L}$ and of the power prediction error $\epsilon_{p_n} = p_n - \hat{p}_{n|n-L}$ are given by

$$\sigma_{\epsilon_c}^2 = r_g - \mathbf{r}_{g\varphi}^H \mathbf{R}_\varphi^{-1}\mathbf{r}_{g\varphi}, \quad (8)$$

$$\sigma_{\epsilon_p}^2 = r_g^2 - |\mathbf{r}_{g\varphi}^H \mathbf{R}_\varphi^{-1}\mathbf{r}_{g\varphi}|^2, \quad (9)$$

respectively. Thus, by (3), (5), (6) and (8), the optimum quadratic power predictor can be expressed in terms of the MSE-optimal linear FIR channel predictor as $\hat{p}_{n|n-L} = |\hat{g}_{n|n-L}|^2 + \sigma_{\varepsilon_c}^2$. The bias compensation will reduce the total prediction MSE and provides superior performance as compared to the use of linear power predictors that are based on channel power samples ($|y_n|^2$) as regressors [11]. For a given prediction $\hat{p}_{n|n-L}$, the conditional power prediction error variance, is given by [11]

$$\sigma_{\epsilon_{p_c}}^2(\hat{p}_{n|n-L}) = \sigma_{\epsilon_c}^2[2\hat{p}_{n|n-L} - \sigma_{\epsilon_c}^2]. \quad (10)$$

If we average over the predicted power in (10), we obtain

$$\sigma_{\epsilon_p}^2 = \sigma_{\epsilon_c}^2[2r_g - \sigma_{\epsilon_c}^2], \quad (11)$$

since, with the unbiased predictor, $E(\hat{p}_{n|n-L}) = E(p_n) = r_g$.

Another indication of the predictor performance is the relative standard deviation of the conditional power prediction error. Using (10), this measure is given by

$$\frac{\sigma_{\epsilon_{p_c}}(\hat{p}_{n|n-L})}{\hat{p}_{n|n-L}} = \sigma_{\epsilon_c}\sqrt{\frac{2\hat{p}_{n|n-L} - \sigma^2_{\epsilon_c}}{\hat{p}^2_{n|n-L}}}. \quad (12)$$

For a given σ_{ϵ_c}, (12) increases as $\hat{p}_{n|n-L}$ becomes small, i.e. when we predict a fading dip.

To solve the rate adaptation optimization problem, the pdf of the instantaneous SNR is required. In section 8 of [11], it is shown that if an optimal unbiased power predictor is used which provides a given $\sigma^2_{\epsilon_c}/r_g$, then the pdf of γ_n conditioned on $\hat{\gamma}_{n|n-L}$ will be

$$\begin{aligned} f_\gamma(\gamma|\hat{\gamma}) =& \frac{\mathrm{U}(\gamma)\mathrm{U}(\hat{\gamma} - \bar{\gamma}\sigma^2_{\epsilon_c}/r_g)}{\bar{\gamma}\sigma^2_{\epsilon_c}/r_g} \exp\left[-\frac{\gamma + \hat{\gamma} - \bar{\gamma}\sigma^2_{\epsilon_c}/r_g}{\bar{\gamma}\sigma^2_{\epsilon_c}/r_g}\right] \\ & \mathrm{I}_0\left(\frac{2}{\bar{\gamma}\sigma^2_{\epsilon_c}/r_g}\sqrt{\gamma(\hat{\gamma} - \bar{\gamma}\sigma^2_{\epsilon_c}/r_g)}\right), \end{aligned} \quad (13)$$

where U( ) is the Heaviside's step function and I_0( ) is the zeroth order modified Bessel function. The time index n was dropped in the pdf expressions since γ_n and $\hat{\gamma}_{n|n-L}$ are both stationary random processes. The pdf of $\hat{\gamma}$ will be given by

$$f_{\hat{\gamma}}(\hat{\gamma}) = \frac{\mathrm{U}(\hat{\gamma} - \bar{\gamma}\sigma^2_{\epsilon_c}/r_g)}{\bar{\gamma}(1 - \sigma^2_{\epsilon_c}/r_g)} \exp\left[-\frac{\hat{\gamma} - \bar{\gamma}\sigma^2_{\epsilon_c}/r_g}{\bar{\gamma}(1 - \sigma^2_{\epsilon_c}/r_g)}\right]. \quad (14)$$

This is a shifted $\chi^2(2)$-distribution, with the shift $\bar{\gamma}\sigma^2_{\varepsilon_c}/r_g$ caused by the bias compensation term in (5).

IV. M-QAM BER Performance

The transmitter adjusts the constellation size and possibly also the transmit power based on the instantaneous predicted SNR $\hat{\gamma}_{n|n-L}$, where the time index n will be dropped in the following. Assuming a square M-QAM with Gray encoded bits, constellation size M_i, and transmit power $S(\hat{\gamma})$, the instantaneous BER as a function of γ and $\hat{\gamma}$ on an AWGN channel, is approximated by [12]

$$\mathrm{BER}(\gamma, \hat{\gamma}) \approx \frac{2\left(1 - \frac{1}{\sqrt{M_i}}\right)}{\log_2 M_i}\mathrm{erfc}\left(\sqrt{\frac{1.5\gamma\frac{S(\hat{\gamma})}{\bar{S}}}{M_i - 1}}\right) \quad (15)$$

which is tight for high SNRs. In [6], it is shown that (15) can be further approximated as

$$\mathrm{BER}(\gamma, \hat{\gamma}) \approx 0.2\exp\left(\frac{-1.6\gamma}{M_i - 1}\frac{S(\hat{\gamma})}{\bar{S}}\right) \quad (16)$$

which is tight within 1 dB for $M_i \geq 4$ and BER$\leq 10^{-3}$. Using (13) to average (16) over the whole range of instantaneous true SNR, γ,

$$\mathrm{BER}(\hat{\gamma}) = \int_0^\infty \mathrm{BER}(\gamma, \hat{\gamma}) f_\gamma(\gamma|\hat{\gamma}) d\gamma, \quad (17)$$

the instantaneous BER as a function of the instantaneous predicted SNR[1], $\hat{\gamma}$, is obtained as

$$\mathrm{BER}(\hat{\gamma}) \approx 0.2 z(\hat{\gamma}) \exp\left[(1 - x(\hat{\gamma}))(1 - z(\hat{\gamma}))\right] \quad (18)$$

where

$$x(\hat{\gamma}) = \frac{\hat{\gamma}}{\bar{\gamma}\sigma^2_{\epsilon_c}/r_g}, \; z(\hat{\gamma}) = \frac{1}{1 + A_i S(\hat{\gamma})}, \; A_i = \frac{1.6}{M_i - 1}\frac{\bar{\gamma}\sigma^2_{\epsilon_c}/r_g}{\bar{S}}. \quad (19)$$

Note that $x(\hat{\gamma}) \geq 1$ since $\hat{\gamma} \geq \bar{\gamma}\sigma^2_{\epsilon_c}/r_g$ by (14) and that $0 \leq z(\hat{\gamma}) < 1$. Finally, similar to [6], the average BER is defined by

$$\overline{\mathrm{BER}} = \frac{\sum_{i=0}^{N-1} k_i \int_{\hat{\gamma}_i}^{\hat{\gamma}_{i+1}} \mathrm{BER}(\hat{\gamma}) f_{\hat{\gamma}}(\hat{\gamma}) d\hat{\gamma}}{\sum_{i=0}^{N-1} k_i \int_{\hat{\gamma}_i}^{\hat{\gamma}_{i+1}} f_{\hat{\gamma}}(\hat{\gamma}) d\hat{\gamma}}. \quad (20)$$

V. Optimal Rate and Power Adaptation

The spectral efficiency of a modulation scheme is given by the average data rate per unit bandwidth (R/B) where R is the data rate and B is the transmitted signal bandwidth. When a modulation with constellation size M_i is chosen, the instanteneous data rate is k_i/T_s (bps). Assuming the Nyquist data pulses ($B = 1/T_s$), the spectral efficiency is given by

$$\eta_B = \frac{R}{B} = \sum_{i=0}^{N-1} k_i \int_{\hat{\gamma}_i}^{\hat{\gamma}_{i+1}} f_{\hat{\gamma}}(\hat{\gamma}) d\hat{\gamma} \quad \text{bps/Hz}. \quad (21)$$

In this work, we consider the following scenarios. First, we intend to maximize the spectral efficiency where both the average power and instantaneous BER are constrained. The transmit power as well as the rate are adapted to satisfy the requirements. Then, we study a case where constant transmit power is presumed. The motivation is that the data rate adaptation has the major effect in increasing the spectral efficiency as compared to the power adaptation [1], [5]. Also, transmission with variable power complicates the practical implementation. We thereafter relax the BER constraint by constrainting the average BER instead of the instantaneous BER, which would be sufficient for most applications such as speech. We illustrate the derivation of the optimum rate region boundaries and possibly also transmit power adjustment of these cases. Once the optimal rate region boundaries are evaluated, the spectral efficiency can easily be obtained according to (21).

A. Instantaneous BER and variable power (I-BER, V-Pow)

The case we consider first is maximizing the spectral efficiency subject to the average transmit power constraint

$$\int_0^\infty S(\hat{\gamma}) f_{\hat{\gamma}}(\hat{\gamma}) d\hat{\gamma} \leq \bar{S} \quad (22)$$

and the instantaneous BER constraint

$$\mathrm{BER}(\hat{\gamma}) = P_b. \quad (23)$$

The constraint (23) together with (18) show that one of the variables, i.e. $z(\hat{\gamma})$ or $x(\hat{\gamma})$, can be expressed in terms of the other. Thus, we take the natural logarithm of (23) based on (18) and then use the Taylor approximation $\ln z(\hat{\gamma}) \approx z(\hat{\gamma}) - 1$ about $z(\hat{\gamma}) = 1$ to obtain

$$z(\hat{\gamma}) \approx 1 - \frac{1}{x(\hat{\gamma})}\ln(0.2/P_b). \quad (24)$$

Using (19) in the above equation, we obtain an expression for the power adjustment within the SNR region for rate i given by

$$S_i(\hat{\gamma}) \approx \left[\frac{\frac{1}{A_i}\frac{\bar{\gamma}\sigma^2_{\epsilon_c}}{r_g}\ln(0.2/P_b)}{\hat{\gamma} - \frac{\bar{\gamma}\sigma^2_{\epsilon_c}}{r_g}\ln(0.2/P_b)}\right]\mathrm{U}\left(\hat{\gamma} - \frac{\bar{\gamma}\sigma^2_{\epsilon_c}}{r_g}\ln(0.2/P_b)\right) \quad (25)$$

[1]This is an average over the pdf over the true instantaneous SNR for one specific modulation scheme. The term *instantaneous* BER refers to the fact that the BER is a function of instantaneous predicted SNR.

where $S_i(\hat{\gamma}) = S(\hat{\gamma})$ when $\hat{\gamma} \in [\hat{\gamma}_i, \hat{\gamma}_{i+1})$. This simplifies the optimization problem to a search for the optimal rate region boundaries. Hence, we form the Langrangian function from the spectral efficiency criterion (21) and the power constraint (22), which is here treated as an equality constraint. It is given by

$$\begin{aligned} J(\hat{\gamma}_0, \hat{\gamma}_1, \cdots, \hat{\gamma}_{N-1}) &= \sum_{i=0}^{N-1} k_i \int_{\hat{\gamma}_i}^{\hat{\gamma}_{i+1}} f_{\hat{\gamma}}(\hat{\gamma}) d\hat{\gamma} \\ &+ \lambda \left(\sum_{i=0}^{N-1} \int_{\hat{\gamma}_i}^{\hat{\gamma}_{i+1}} S_i(\hat{\gamma}) f_{\hat{\gamma}}(\hat{\gamma}) d\hat{\gamma} - \bar{S} \right) \end{aligned} \tag{26}$$

where $\lambda \neq 0$ is the Lagrangian multiplier. Solving

$$\frac{\partial J}{\partial \hat{\gamma}_i} = 0, \quad 0 \leq i \leq N-1 \tag{27}$$

results in

$$S_{i-1}(\hat{\gamma}_i) - S_i(\hat{\gamma}_i) = \frac{k_i - k_{i-1}}{\lambda}, \quad 0 \leq i \leq N-1 \tag{28}$$

where $k_{-1} = 0$ and $S_{-1}(\hat{\gamma}) = 0$. From (25) and (28), we obtain

$$\hat{\gamma}_i = \ln\left(\frac{0.2}{P_b}\right)\left(\frac{\bar{\gamma}\sigma^2_{\epsilon_c}}{r_g} - \frac{\bar{S}}{1.6}\frac{M_i - M_{i-1}}{k_i - k_{i-1}}\lambda\right), 0 \leq i \leq N-1. \tag{29}$$

The Lagrange multiplier λ is numerically evaluated based on the power constraint (22).

B. Instantaneous BER and constant power (I-BER, C-Pow)

We now consider the use of an instantaneous BER constraint and of a constant transmit power $S(\hat{\gamma}) = S$ that is adjusted to satisfy the average power constraint (22) with equality. The BER expression (18) then becomes

$$\mathrm{BER}(\hat{\gamma}) = \frac{0.2}{1 + A_i S} \exp\left[\frac{A_i S}{1 + A_i S}\left(1 - x(\hat{\gamma})\right)\right]. \tag{30}$$

The average power constraint (22), implies that the transmit power used when transmission does occur will be higher than $\bar{S}$ given by

$$S = \bar{S} \exp\left[\frac{\hat{\gamma}_0 - \bar{\gamma}\sigma^2_{\epsilon_c}/r_g}{\bar{\gamma}(1 - \sigma^2_{\epsilon_c}/r_g)}\right]. \tag{31}$$

Moreover, the instantaneous BER constraint must be fulfilled at all the rate region boundaries such that

$$\mathrm{BER}(\hat{\gamma}) \leq \mathrm{BER}(\hat{\gamma}_i) = P_b, \quad \hat{\gamma} \in [\hat{\gamma}_i, \hat{\gamma}_{i+1}), \quad 0 \leq i \leq N-1 \tag{32}$$

which by (30) and (19) results in

$$\hat{\gamma}_i = \frac{\bar{\gamma}\sigma^2_{\epsilon_c}}{r_g}\left[1 - \frac{1 + A_i S}{A_i S}\ln\left(\frac{P_b}{0.2}\left(1 + A_i S\right)\right)\right], 0 \leq i \leq N-1. \tag{33}$$

The cut-off SNR $\hat{\gamma}_0$ and the transmit power S are found through (31) and (33). Then, $\{\hat{\gamma}_i\}_{i=1}^{N-1}$ are easily obtained from (33).

C. Average BER and constant power (A-BER, C-Pow)

Finally, we investigate the case concerning the average BER constraint with constant transmit power. Similar to Section V-B, the transmit power must satisfy (31). The average BER constraint is given by

$$\overline{\mathrm{BER}} \leq P_b, \tag{34}$$

where (30) is used for the instantaneous BER in (20). Forming the Lagrangian function from the criterion (21) and the constraint (34), here treated as an equality constraint, gives

$$\begin{aligned} J(\hat{\gamma}_0, \hat{\gamma}_1, \cdots, \hat{\gamma}_{N-1}) = \sum_{i=0}^{N-1} k_i \int_{\hat{\gamma}_i}^{\hat{\gamma}_{i+1}} f_{\hat{\gamma}}(\hat{\gamma}) d\hat{\gamma} + \\ \lambda\left(\sum_{i=0}^{N-1} k_i \int_{\hat{\gamma}_i}^{\hat{\gamma}_{i+1}} \left(\mathrm{BER}(\hat{\gamma}) - P_b\right) f_{\hat{\gamma}}(\hat{\gamma}) d\hat{\gamma}\right). \end{aligned} \tag{35}$$

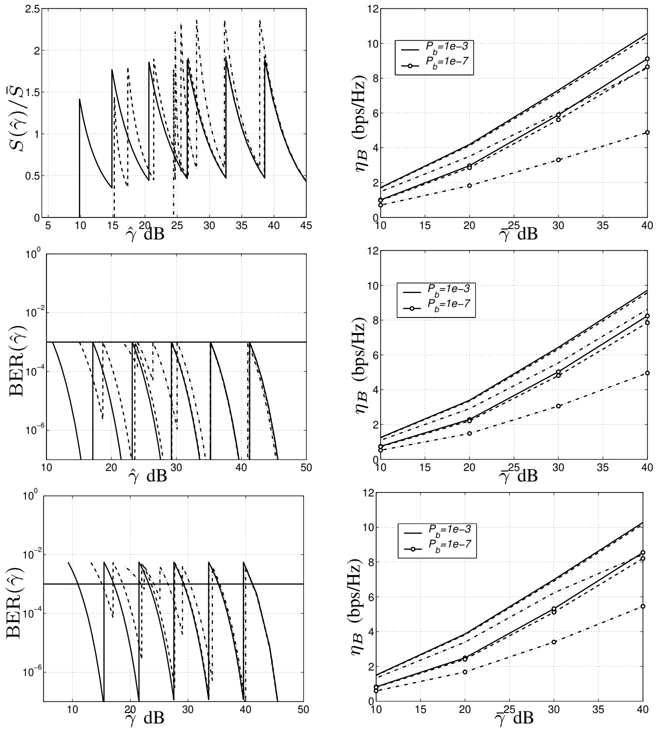

Fig. 2. The plots on the first, second and third rows correspond to the *I-BER, V-Pow* policy, *I-BER, C-Pow* policy and *A-BER, C-Pow* policy, respectively. All the righthand side plots show the M-QAM spectral efficiencies of respective policies for $P_b = 10^{-3}$ and 10^{-7}. The first lefthand side plot illustrates the optimum normalized transmit power and the second and third ones show the instantaneous BER of M-QAM schemes for $\bar{\gamma} = 30$ dB and $P_b = 10^{-3}$. In all the plots, the solid, dashed and dashed-dotted lines correspond to $\sigma^2_{\epsilon_p} = 0.001$, 0.01 and 0.1, respectively.

The optimum rate region boundaries are found through solving (27) which results in

$$\mathrm{BER}(\hat{\gamma}_i) = P_b - \frac{1}{\lambda}, \quad 0 \leq i \leq N-1. \tag{36}$$

Similar to the previous case, we have

$$\hat{\gamma}_i = \frac{\bar{\gamma}\sigma^2_{\epsilon_c}}{r_g}\left[1 - \frac{1 + A_i S}{A_i S}\ln\left(\frac{P_b - \frac{1}{\lambda}}{0.2}(1 + A_i S)\right)\right], \tag{37}$$

for $0 \leq i \leq N-1$. We can evaluate the optimal rate region boundaries and transmit power through (31) and (37) based on λ that satisfies the average BER constraint.

VI. Results

We assume that six M-QAM signal constellations corresponding to 4-QAM, 16-QAM, 64-QAM, 256-QAM, 1024-QAM and 4096-QAM, are available at the transmitter. Also, a flat Rayleigh fading channel with $r_g = 1$ is presumed. The optimal region boundaries for different policies when the required BER is $P_b = 10^{-3}$, the

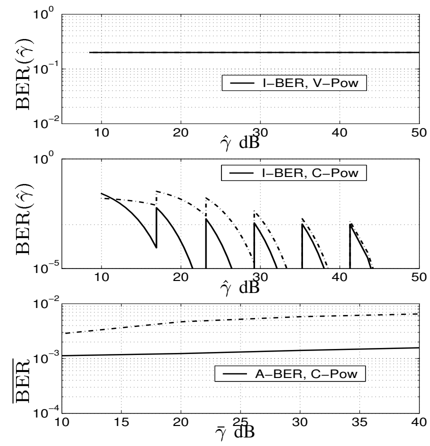

Fig. 3. The effect of imperfect CSI on the BER performace of M-QAM schemes. The results are shown for the three considered transmission schemes when the prediction uncertainty is not taken into account in the rate and power adaptation, for a required $P_b = 10^{-3}$. For the *I-BER, V-Pow* and *I-BER, C-Pow* policies, $\bar{\gamma} = 30$ dB. The solid and dashed-dotted lines correspond to $\sigma^2_{\epsilon_p} = 0.01$ and 0.1, respectively.

prediction error variances are $\sigma^2_{\epsilon_p} = 0.001$, 0.01 and 0.1 and the average received SNR is $\bar{\gamma} = 30$ dB, can be seen from Fig. 2. Here, $\sigma^2_{\epsilon_p} = 0.1$, corresponds to a prediction of 0.33 wavelengths ahead in space for a 1900 MHz carrier at 50 Km/h vehicle speed [10]. It is shown that for the *I-BER, V-Pow* policy, the transmit power follows the inverse water-filling pattern w.r.t. $\hat{\gamma}$ within each rate region interval. The peak power within each interval increases as the rate increases. Under the *I-BER, C-Pow* policy, the instantaneous BER does not exceed the required BER while it reaches the target BER at the boundaries as intended. Finally, the *A-BER, C-Pow* policy results in an instantaneous BER fluctuation around the required average BER to maintain the target BER, on average.

An interesting observation here is the effect of the prediction error variance on the rate region boundaries. We see that for a large prediction error variance, the boundaries are *raised* for SNRs lower than the average SNR. To a less extent, they are usually *lowered* for SNRs higher than the average SNR. A reasonable explanation is that when we predict into a fading dip (low SNR), the *relative* conditional prediction standard deviation will become larger. This will contribute to making the scheme cautious when entering fading dips. As the prediction error is increased, the scheme would not transmit at all during an increasing fraction of the time (when $\hat{\gamma}$ is below $\bar{\gamma}$). Due to the average power constraint, this allows the use of higher transmit power when transmission is allowed. For $\hat{\gamma} >> \bar{\gamma}$, the last effect sometimes dominates. This explains why the SNR limits for use of the largest constellation size can be reduced.

The maximum spectral efficiency for $P_b = 10^{-3}$ and 10^{-7}, and $\sigma^2_{\epsilon_p} = 0.001$, 0.01 and 0.1 are also illustrated in Fig. 2 for the studied policies. The similar trend observed in these plots is that the gain in the spectral efficiency when using good predictors is considerable as compared to the poor predictors. Comparing different policies from the spectral efficiency point of view, we see that for small prediction error variance, the highest and lowest spectral efficiencies are provided by *I-BER, V-Pow* and *I-BER, C-Pow*, respectively. However, as the predictor deteriorates, the spectral efficiencies of all the policies become closer to each other.

Finally, Fig. 3 is shown to highlight the importance of considering realistic assumptions for the design. In this example, the adaptive modulation systems are designed under the assumption of perfect channel prediction. But, the transmitted signals are experiencing different channels than the predicted ones due to the inaccurate predictions. It is evident that the target BER will no longer be attained by neglecting the prediction errors in the design.

VII. Conclusion

The optimum design of an adaptive modulation scheme based on uncoded M-QAM modulation is investigated. The transmitter adjusts the transmission rate and possibly also power based on the predicted SNR to maximize the spectral efficiency while satisfying the BER and average transmit power constraints.

Optimum solutions for adjusting the adaptive rate and transmit power are derived. The analytical results show that when the prediction error increases, the rate region boundaries for a given constellation size are raised for the SNRs lower than average SNR, while they are sometimes lowered for the SNRs higher than the average SNR. Moreover, the spectral efficiency decreases as the predictor error variance increases. This effect is more noticable when the required BER decreases, as expected. Also, the gain due to the transmission with varying power is minor and becomes even negligible when the prediction quality deteriotes. It is demonstrated that the QoS considerably degrades when the system is not designed based on realistic assumptions such as erroneous prediction.

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