Transmit antenna diversity in Ricean fading MIMO channels with co-channel interference

Mattias Wennström, Tommy Öberg

Signals and Systems Group, Uppsala University,
Box 528, SE-751 20 Uppsala, Sweden, Email:mw@signal.uu.se

ABSTRACT

Transmit antenna diversity using closed loop and open loop modes are compared in environments with line of sight propagation and spatially and temporally non-white noise. The connection between the feedback channel capacity in closed loop modes and the channel coherence time is investigated. It is shown that in LOS environments, the requirements on the feedback channel bit rate is lower. Furthermore, we derived the beamforming vector in closed loop transmit diversity with directional interference and showed how this gives an equivalent gain in SNR of about 3 dB at the receiver, even at a beamforming weight phase resolution of 90°.

INTRODUCTION

When communicating over a flat fading channel, some kind of diversity is usually required to limit the required transmit power and to reduce co-channel interference. A bandwidth efficient way to introduce diversity is to use several transmit and/or receive antennas. To use antenna diversity at the receiver is straightforward, because the channel can be estimated from the received data. To use transmit antenna diversity is, however, more cumbersome because the channel is not readily available at the transmitter. This problem can be approached in several ways; a simple way is to design the system to use time-division duplex (TDD) with a duplex distance smaller that the channel coherence time. Hence, the uplink channel is valid as an estimate of the downlink channel and knowledge of the downlink channel is then used to optimize the downlink transmission. If a frequency division duplex (FDD) system is used (or a TDD system with duplex delay larger than the coherence time) , the duplex distance is commonly much larger than the channel coherence bandwidth and the uplink data cannot be used to optimize the downlink transmission. Instead, one can decide to use an open-loop transmit diversity, as was presented by Tarokh et.al. [1], introducing the *space-time codes* (STC). One can also design the system with a feedback channel, as in the currently developing W-CDMA standard. The feedback signalling provides the transmitter with information on how to select the transmission to minimize the bit error rate of the transmission. The feedback channel has a low bandwidth and to decide what kind of information to send on this channel is an area currently under investigation. In feedback systems, the transmitter side has only partial knowledge of the channel, and the reliability of this knowledge depends on the feedback rate and how fast the channel changes, i.e. the channel time-frequency product. The time-frequency product is $T_s f_D$, where T_s is the symbol time and f_D is the Doppler frequency.

In [2], the side information was modeled using a purely statistical approach and it was shown how a space-time code (open-loop approach) can be improved when the transmitter has partial knowledge of the channel.

In this paper, we compare the performance of the open loop space-time block codes with a variable bit rate feedback closed-loop transmit diversity system and also with a TDD system, which corresponds to a system with perfect channel feedback. We are interested in how these systems vary in different radio environments, and investigate how the required bit rate of the feedback channel varies with the temporal characteristics of the channel. We add the effect of a line of sight (LOS) component in the channel and also study how a co-channel interferer affect the performance. As MIMO channel measurements have shown [3], the STC concept shows promising results in a MIMO channel even where line of sight (LOS) components exists between TX and RX antennas.

CHANNEL MODEL

Assume that the flat fading MIMO channel is represented by the $N_T \times N_R$ matrix $\mathbf{H}$:

$$\mathbf{H} = \begin{pmatrix} h_{11} & h_{12} & \cdots & h_{1N_R} \\ h_{21} & h_{22} & \cdots & h_{2N_R} \\ \vdots & \vdots & & \vdots \\ h_{N_T1} & h_{N_t2} & \cdots & h_{N_TN_R} \end{pmatrix} \quad (1)$$

The received baseband signal can be written as

$$\mathbf{x}(k) = \mathbf{H}\mathbf{c}(k) + \mathbf{n}(k) \quad (2)$$

where $\mathbf{x}(k)$ is the $N_R \times 1$ received vector, $\mathbf{c}(k)$ is the transmitted vector and $\mathbf{n}(k)$ is the receiver noise plus interference with covariance matrix $\mathbf{R}_{nn} = E\left\{\mathbf{n}(k)\mathbf{n}^H(k)\right\}$. or in block form

$$\mathbf{X} = \mathbf{HC} + \mathbf{N} \tag{3}$$

where $\mathbf{X}$ and $\mathbf{N}$ is $N_R \times P$ and $\mathbf{C}$ is $N_T \times P$. Hence we have assumed that the channel is constant over P symbols.

To model the wireless channel with different degree of scattering richness, we use the well known Ricean model [4]. The channel matrix are decomposed into the direct (LOS) component that illuminates the arrays entirely and the scattered, Rayleigh distributed component. The Ricean K-factor, defined as the ratio of deterministic (LOS) and scattered power is introduced and gives pure LOS or pure scattering as extreme cases ($K = \infty$ and $K = 0$).

$$\begin{aligned}&\mathbf{H} = \mathbf{H}_{LOS} + \mathbf{H}_{scat} = \\ &\sqrt{\frac{KG}{K+1}}\mathbf{a}^{TX}(\theta_{TX})(\mathbf{a}^{RX}(\theta_{RX}))^H + \sqrt{\frac{G}{K+1}}\tilde{\mathbf{H}}_{scat}\end{aligned} \tag{4}$$

Here θ is the respective angle from array broadside and $\mathbf{a}(\theta)$ is the normalized array response vector. G is the large scale path gain including the antenna element gains.

The matrix $\mathbf{H}_{scat}$ is calculated using a scattering disc model. It is a simple, yet detailed channel model that includes the spatial dimension. It was presented in [5] and consists of a circular disc of uniformly distributed scatterers placed around the mobile station. The radius of the disc is easily varied to model different scenarios. The path gain (elements in $\mathbf{H}_{scat}$) are written as

$$\begin{aligned}&H^{i,j}_{scat} = \\ &= \sum_{l=1}^{L} \alpha_l \exp\left\{jk(r_{t_j\to s_l} + r_{s_l\to r_i})\right\} g_{t_j}(\phi_l^T) g_{r_i}(\phi_l^R),\end{aligned} \tag{5}$$

where the number of scatterers is L and α_l is a complex Gaussian distributed reflection coefficient with zero mean and unit variance. Furthermore, $r_{t_j\to s_l}$ and $r_{s_l\to r_i}$, denotes the distance from mobile antenna j to scatterer j and scatterer j to base station antenna i respectively. Finally $g_{t_j}(\phi_l^R)$ and $g_{r_i}(\phi_l^R)$ are the complex voltage radiation patterns of the antennas.

By assigning the mobile a velocity and a direction through the cloud of scatterers, we can generate a time varying channel for each transmit-receive antenna pair with different characteristics by varying the disc radius, speed of mobile, LOS parameter K or the base to mobile distance. Note that this model generates correlated fading between the signals from the receiving antennas.

DESCRIPTION OF METHODS

In this section the open and closed loop methods are described. First, we show the expression for the STC bit error rate probability (BER) and then we assumed that the transmitter has perfect knowledge of the channel and derive the corresponding beamforming vector. When the channel is perfectly known we get a performance benchmark, with the lowest BER possible.

Assume that the codewords in the STC are of length P and that the channel is stationary during the time of transmission of one codeword. Form the vectors $\mathcal{X} = \text{vec}(\mathbf{X})$, $\mathcal{C} = \text{vec}(\mathbf{C})$, $\mathcal{N} = \text{vec}(\mathbf{N})$ and the matrix $\mathcal{H} = \mathbf{I}_P \otimes \mathbf{H}$, where $\otimes$ is the Kronecker product. Now, write the detection hypothesis as:

$$\begin{array}{ll} H_0: & \mathcal{X} = \mathcal{H}\mathcal{C}_0 + \mathcal{N} \\ H_1: & \mathcal{X} = \mathcal{H}\mathcal{C}_1 + \mathcal{N} \end{array} \tag{6}$$

The likelihood ratio test gives the test statistic $\mathcal{T}$

$$\begin{aligned}&\mathcal{T} = \frac{p(\mathcal{X}|H_1)}{p(\mathcal{X}|H_0)} \sim \\ &\mathcal{N}\left(-\frac{1}{2}\|\mathcal{R}^{-1/2}\mathcal{H}(\mathcal{C}_1 - \mathcal{C}_0)\|^2, \|\mathcal{R}^{-1/2}\mathcal{H}(\mathcal{C}_1 - \mathcal{C}_0)\|^2\right)\end{aligned} \tag{7}$$

where $\mathcal{R} = E\left(\mathcal{N}\mathcal{N}^H\right)$ and by using the Gaussian tail approximation, and assuming that the code words have the same norm, we can write the probability that the transmitted $N_T \times P$ code matrix $\mathbf{C}_1$ is detected as the other code matrix $\mathbf{C}_0$, i.e. makes an error, as

$$\begin{aligned}&Pr\left\{\mathcal{C}_1 \to \mathcal{C}_0|\mathcal{H},\mathcal{R}\right\} \leq \\ &\qquad \frac{1}{2}\exp\left(-\|\mathcal{R}^{-1/2}\mathcal{H}(\mathcal{C}_1 - \mathcal{C}_0)\|_F^2/4\right)\end{aligned} \tag{8}$$

where we have conditioned this error on the channel matrix and the received interference plus noise covariance matrix, which we assume is known at the transmitter. If the noise is temporally white, we can write $\mathcal{R}^{-1/2} = \mathbf{I}_P \otimes \mathbf{R}_{nn}^{-1/2}$ and (8) reduces to

$$\begin{aligned}&Pr\left\{\mathbf{C}_1 \to \mathbf{C}_0|\mathbf{H},\mathbf{R}_{nn}\right\} \leq \\ &\qquad \frac{1}{2}\exp\left(-\|\mathbf{R}_{nn}^{-1/2}\mathbf{H}(\mathbf{C}_1 - \mathbf{C}_0)\|_F^2/4\right)\end{aligned} \tag{9}$$

This is assumed in the rest of this paper. In (9), $\|\cdot\|_F$ is the Frobenius norm.

A linear transformation

To find the optimal codeword matrix $\mathbf{C}$ that minimizes (9), given $\mathbf{R}_{nn}$ and $\mathbf{H}$, we must perform an exhaustive search over all possible codewords. Instead, we fix the codewords to be *orthogonal space time block codes* (STBC) [6], which have the nice property that the receiver algorithm have very low complexity and that the

Euclidean distance between two codeword matrices is equal for all codeword matrix pairs (denoted μ). In [2], it was shown that the suboptimal transmitted codeword matrix $\mathbf{C}$ can be linearly transformed by a weighting matrix $\mathbf{W}$

$$\tilde{\mathbf{C}} = \mathbf{W}\mathbf{C} \tag{10}$$

to improve the error probability (9). Now, as $\|AQ\|_F^2 = \|A\|_F^2$, when Q is orthogonal, we rewrite equation (9) in the case of orthogonal STBC with transmit weighting as

$$Pr\left\{\tilde{\mathbf{C}}_1 \to \tilde{\mathbf{C}}_0 | \mathbf{H}, \mathbf{R}_{nn}, \mathbf{W}\right\} \leq \frac{1}{2}\exp\left(-\mu^2 \|\mathbf{R}_{nn}^{-1/2}\mathbf{H}\mathbf{W}\|_F^2/4\right) \tag{11}$$

which is to be minimized by proper choice of $\mathbf{W}$, under the power constraint $\|\mathbf{W}\|_F^2 = N_R$. It is straightforward to show that (11) is minimized by the matrix

$$\mathbf{W}_o = \sqrt{N_R}\left[\begin{array}{cccc} \mathbf{v}_{max} & \mathbf{0} & \cdots & \mathbf{0} \end{array}\right] \tag{12}$$

where $\mathbf{v}_{max}$ is eigenvector corresponding to the largest eigenvalue of the matrix $\mathbf{H}^H\mathbf{R}_{nn}^{-1}\mathbf{H}$. This can be interpreted as beamforming in direction of $\mathbf{v}_{max}$ [2]. Note that the optimal codeword matrix now reduces to a rank one matrix. Hence, when the channel is perfectly known at the transmitter, beamforming is the optimal transmission strategy. When there is no knowledge of the channel at the transmitter, the so called open loop scenario, it was shown in [7] that $\mathbf{W} = \mathbf{I}$ minimizes (11) so the STBC is used directly.

Feedback of channel information

We assume that the feedback channel has a capacity of R_{fb} bits per second and that the number of transmit antennas $N_T = 2$. Furthermore, we assume that channel estimation is performed by simultaneously transmitting independent sequences from the transmitter antennas and that the channel is known/predicted perfectly at the receiver. The receiver calculates the eigenvector $\mathbf{v}_{max}$, in (12), corresponding to the largest eigenvalue. Because of limited feedback, we will use

$$\mathbf{v}'_{max} = \left[\begin{array}{c} 1 \\ e^{j\theta} \end{array}\right]\frac{1}{\sqrt{2}} \tag{13}$$

instead, where θ is a uniform quantization of the ideal value of the difference in the arguments of the elements of $\mathbf{v}_{max}$. The approach here is similar to the one used in the W-CDMA standard. The transmitter averages the received feedback signals over two consecutive received bits while maintaining the transmitted power in the both antennas. This can be described as [8]

$$\theta(t) = \arg\left\{i^{t \bmod 2}\mathrm{sgn}\left(z(t)\right) + i^{(t-1) \bmod 2}\mathrm{sgn}\left(z(t-1)\right)\right\} \tag{14}$$

where $z(t)$ is the feedback bit received at time t. The argument $\theta(t)$ have four states similar to QPSK. In [8], a more advanced method is proposed, where more than the two recent samples are used in updating $\theta(t)$. This is beneficial in environments where the fading is varying slowly so the channel is approximately constant over a longer time than it takes to receive two bits in the feedback channel. For optimal performance, the filter length should be updated adaptively to match the channel coherence time. This has nor been investigated further in this paper.

SIMULATION RESULTS

The following parameters were common for all simulations; the number of transmit and receive antennas $N_T = N_R = 2$, antenna separation transmitter was 2λ and at the receiver 0.5λ. The transmitter to receiver distance was 600λ and the radius of the scattering disc 250λ. 30000 samples were collected in each simulation and Alamouti's scheme for the STBC was implemented [9]. The carrier frequency was 2.15 GHz.

The effect of feedback bit rate

We investigated the effect of feedback bit rate in channels with different coherence times. As suggested in [8], we could improve the system performance in slowly varying channels by filtering the feedback bits with an adaptive filter length. The simulations show , for a given feedback rate, at which time-frequency product the BER is higher than the open-loop, or STBC BER, i.e. when the feedback data should not be taken into consideration. In these simulations we have no co-channel interferer, hence $\mathbf{R}_{nn} = \sigma_n^2\mathbf{I}$, where σ_n^2 is the spatially and temporally white noise variance. Figure 1 shows how the BER decreases when the feedback bit rate increases. For the slowly fading channel $T_s f_D = 0.0001$, we perform better than the open-loop STBC at a modest feedback rate of 150 bps. When the channel time coherence is reduced, the feedback rate has to be increased to perform better than the STBC. In the figure the TDD mode is also shown, where we have assumed that the channel state information is perfectly known at the transmitter and hence, the beamforming weight are optimal.

In Figure 2, there exists a strong line of sight component between the basestation and the mobile with $K = 10000$. We see that in this case, a low feedback bit rate will be sufficient as the channel has a clear directional property. Once the direction to the mobile has been accommodated, the transmission can lock on to the mobile and beam-form in that direction. Hence, there is no need for a high bit rate feedback channel.

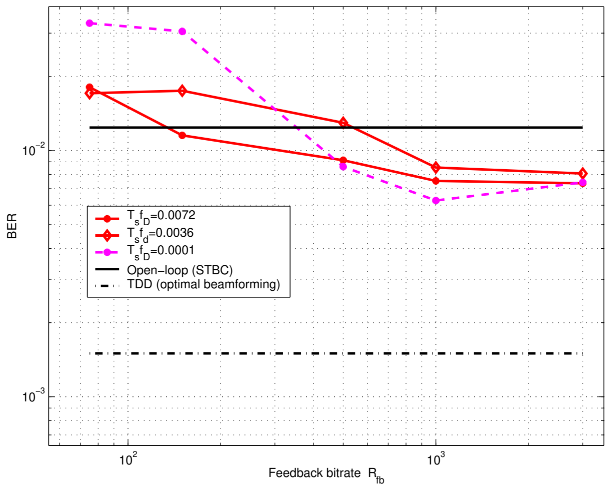

Figure 1: BER when bit rate of feedback channel is varied in channels with different time-frequency products. The TDD and STBC levels are also indicated. SNR=4 dB. No LOS component (K=0).

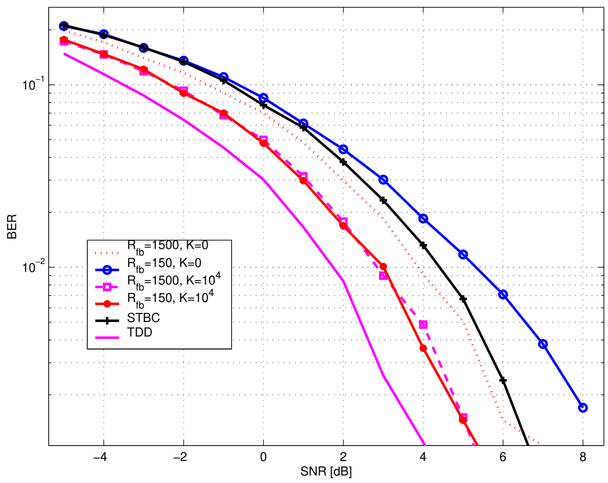

Figure 2: BER as a function of SNR per receive antenna. The LOS component is varied and also the feedback bit rate. Note that in a strong LOS environment, there is no improvement by increasing the feedback bit rate above 150 bps.

Effects of co-channel interference

Another mobile station is now added at the location 800λ from the primary base station. We assume that this base station transmits on the same physical channel as our mobile under study. This implies that the covariance matrix of the noise vector $\mathbf{R}_{nn}$ is a full matrix. We want to investigate the effect of taking the non-spatially white noise into account, by using according to equation (11). We generate the channel using the same model as above, and estimate the noise covariance matrix using the received data. We then use a semi-analytical approach and calculate an upper bound of the error probability using

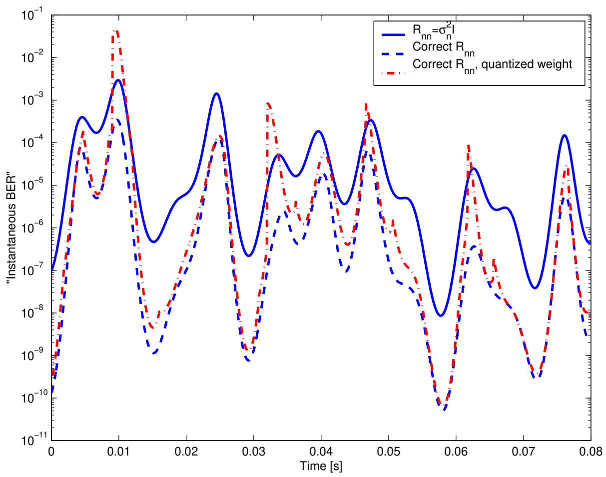

Figure 3: "Instantaneous" BER as a function of time with and without the use of the covariance matrix of the spatially non-white noise

(11) and compare the BER with (11) using $\mathbf{R}_{nn} = \sigma_n^2\mathbf{I}$. Figure 3 shows the instantaneous BER over time when $SNR = 4$ dB without a LOS component. The signal to interference ratio at the mobile is -20 dB. We have also added the BER using the quantized beamforming vector, with 90 degree phase resolution, and we see that on average is the BER lower than the BER acquired by assuming $\mathbf{R}_{nn} = \sigma_n^2\mathbf{I}$.

In Figure 4 the "instantaneous" BER averaged over 3 seconds is shown. The use of the noise covariance matrix corresponds to an equivalent 4 dB gain i SNR and the use of quantized phase reduces the gain to 3.2 dB, still a significant improvement.

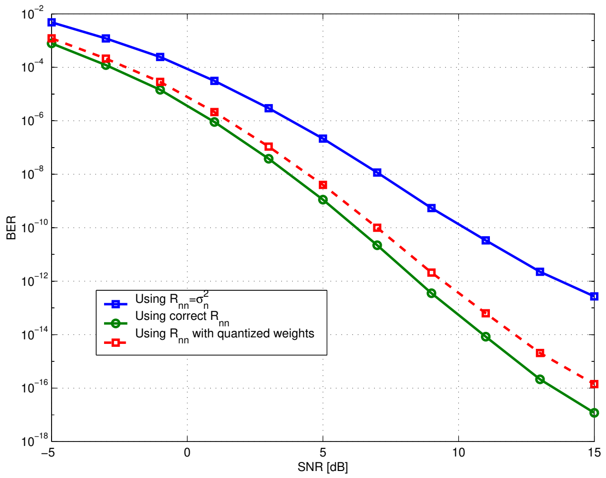

Figure 4: BER as a function of SNR with and without the use of the covariance matrix of the spatially non-white noise

DISCUSSION

We have shown how the feedback information in a transmit diversity system can be adjusted to deal with non-white noise or interferers. This is a common situation in systems with cochannel interference. It was shown that in a spatially colored, but temporally white noise environment, 3.2 dB could be gained in SNR for a 2×2 MIMO system, when a 20 dB weaker interfering base station is present. A simple feedback scheme, similar to the scheme in W-CDMA was compared to an open-loop solution, using space time block codes and it was shown how the required feedback bit rate depends on the channel time-frequency product. In LOS channels, the demands on the feedback channel capacity becomes smaller. In future work, the feedback signalling can be made adaptive, to provide the transmitter with the best possible information, independent on the mobility of the mobile user. Further studies should also deal with more than 2 transmitting antennas, as the demands on the feedback bit rate then increases.

REFERENCES

[1] V.Tarokh, A.Naguib, N.Seshadri, and A.R.Calderbank, "Space-time codes for high data rate wireless communications: performance criteria," in *IEEE ICC Conference*, June 1997.

[2] G.Jöngren and B.Ottersten, "Combining transmit antenna weights and orthogonal space-time block codes by utilizing side information," in *Signals, Systems, and Computers, 1999. Conference Record of the Thirty-Third Asilomar Conference*, USA, 1999, pp. 1562–1566.

[3] D.P. McNamara, M.A. Beach, P. Karlsson, and P.N.Fletcher, "Initial characterisation of multiple input multiple output (mimo) channels for space time communication," in *IEEE VTC 2000 Conference*, Boston, MA, USA, 2000.

[4] S.Rice, "Mathematical analysis of random noise," *Bell System Technical Journal*, vol. 23, 1944.

[5] P.Petrus, J.H.Reed, and T.S.Rappaport, "Geometrically based statistical channel model for macrocellular mobile environments," in *Proc. IEEE Global Telecommunications Conference*, 1996.

[6] V.Tarokh, H.Jafarkhani, and A.R.Calderbank, "Space-time block codes from orthogonal designs," *IEEE Transactions on Information Theory*, vol. 45, pp. 1456–1467, 1999.

[7] George Jöngren, Mikael Skoglund, and Björn Ottersten, "Combining transmit beamforming and orthogonal space-time block codes by utilizing side information," in *Proceedings IEEE Sensor Array and Multichannel Signal Processing Workshop*, Boston, USA, March 2000.

[8] A.Hottinen and R.Wichman, "Transmit diversity using filtered feedback weights in the FDD/WCDMA system," in *Broadband Communications, 2000. Proceedings.*, Zurich, 2000, pp. 15–21.

[9] S.M. Alamouti, "A simple transmit diversity technique for wireless communications," *IEEE Journal on selected areas in communications*, vol. 16, pp. 1451–1458, 1998.