# Tracking of time-varying Mobile Radio Channels with WLMS Algorithms:

# A Case study on D-AMPS 1900 Channels

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# Outline

WLMS (Wiener LMS):

- novel design of adaptation laws with constant gains
- prediction, filtering and fixed-lag smoothing
- close to optimal Kalman performance, but
- much lower complexity

Example of MSE tracking performance and complexity.

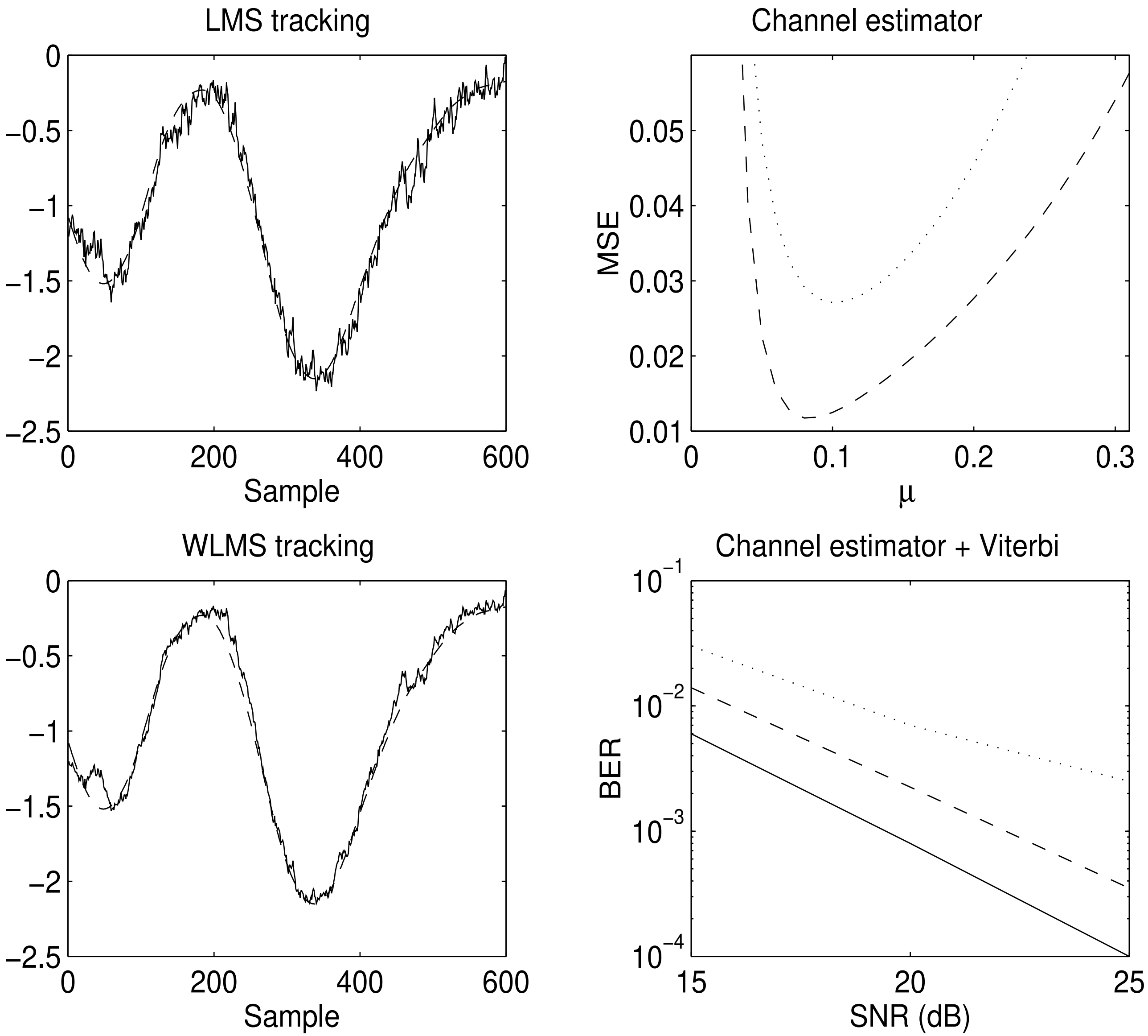
Two-tap fast fading channel. (See paper 4 in session 1.01):

| | Input (symbol) properties | Kalman | WLMS | LMS | RLS |
|---|---|---|---|---|---|
| | White and constant modulus: | 0.010 | 0.011 | 0.020 | 0.026 |
| | White and Gaussian: | 0.012 | 0.015 | 0.032 | 0.038 |
| | Colored and Gaussian: | 0.026 | 0.038 | 0.085 | 0.075 |
| # of add/mult | White inputs: | 214 | 30 | 18 | 72 |
| | Colored inputs: | 214 | 44 | 18 | 72 |

LMS and RLS are here inadequate

- Assume channel taps to have (coupled) ARIMA statistics.
- Filters are introduced into LMS, and tuned via an iterative Wiener procedure.
- An exact MSE tracking expression is used in the tuning.
- The design is here evaluated on fast fading channels in IS-136.
- It is used in decision-directed mode together with Viterbi detectors, in which prediction estimates are required.

# An illustration




dotted - LMS
dashed - WLMS
solid - correct channel

# The Channel Model

Time-varying linear regression:

Measurement, Regressor (symbols), Channel coefficients, Noise

$$y(n) = \underbrace{\begin{bmatrix} u(n) & u(n-1) & \dots & u(n-\mathrm{M}+1) \end{bmatrix}}_{\varphi^*(n)} \underbrace{\begin{bmatrix} h_0(n) \\ \dots \\ h_{\mathrm{M}\text{-}1}(n) \end{bmatrix}}_{h(n)} + v(n)$$

Autocorrelation matrices

$$\mathrm{R} = E\{\varphi(n)\varphi^*(n)\} \quad (\mathrm{R} = \sigma_u^2 \mathrm{I})$$

$$\mathrm{R_h} = E\{h(n)h^*(n)\}$$

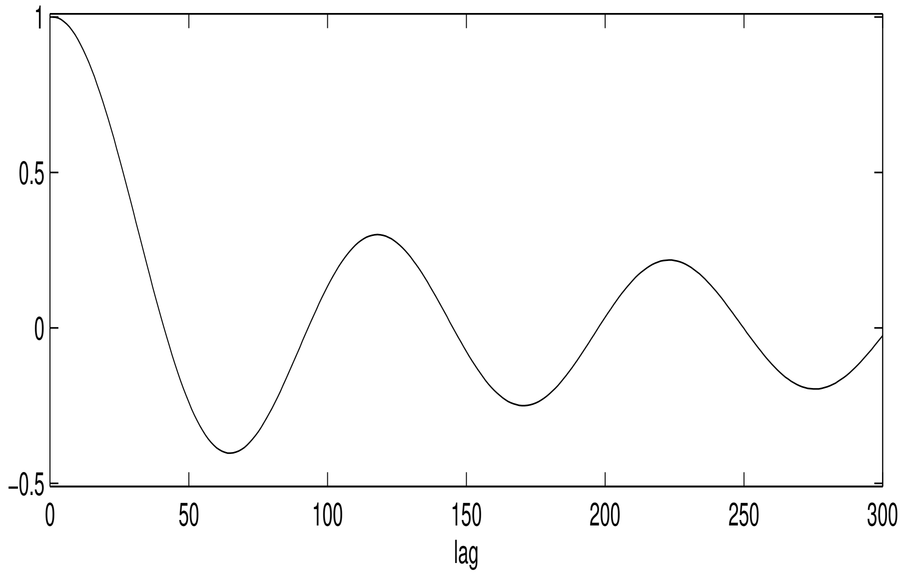
Classical fading spectrum

$$\phi_h(\Omega) = \frac{2}{\sqrt{\Omega_D^2 - \Omega^2}} \mathrm{R_h}, \ |\Omega| < \Omega_D$$

$\longleftrightarrow$

Noise variance $\sigma_v^2$

Covariance function of $h_i(n)$ $(\Omega_D = 0.06)$



# The channel estimator

One possible implementation of the WLMS algorithm

The algorithm recursions:

$$\varepsilon(n) = y(n) - \varphi^*(n)\hat{h}(n|n-1)$$

$$\hat{h}(n|n) = \hat{h}(n|n-1) + \mu R^{-1}\varphi(n)\varepsilon(n)$$

$$\hat{h}(n+k|n) = P_k(q^{-1})\hat{h}(n|n)$$

Coefficient smoothing-prediction filters

In **WLMS**: $P_k(q^{-1}) = \dfrac{Q_k(q^{-1})}{Q_0(q^{-1})}I$

k < 0: Smoothing
k > 0: Prediction

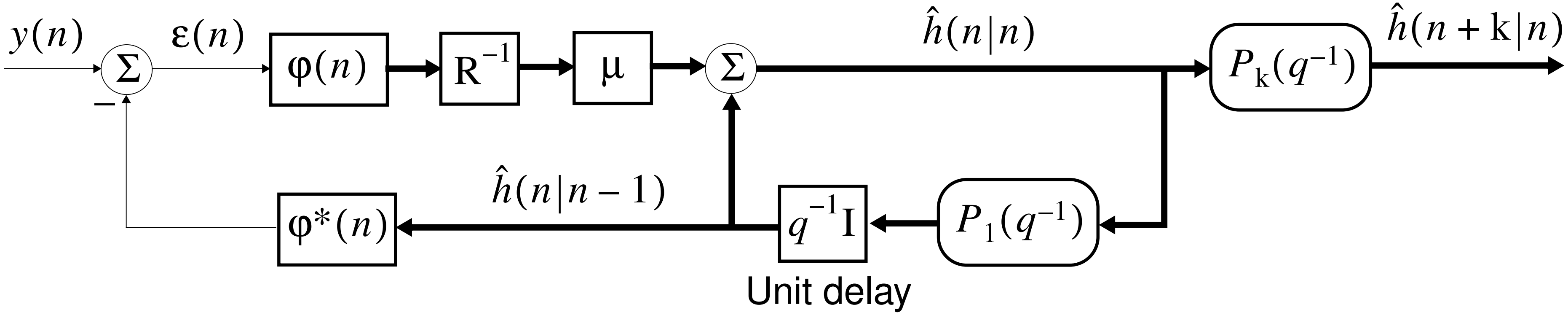
In **LMS**

$$P_k(q^{-1}) = I$$
$$R^{-1} = I$$

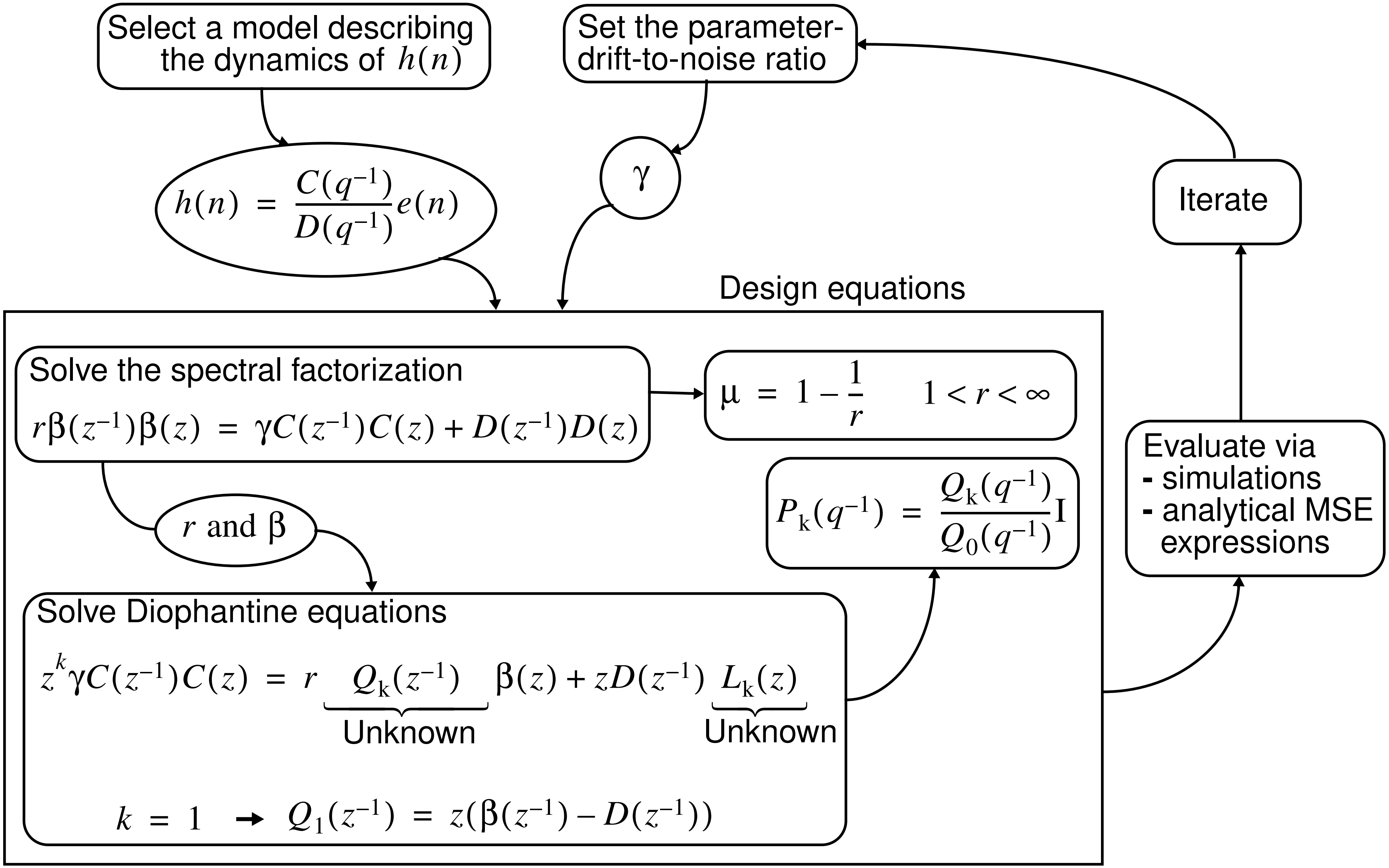
Minimize the steady-state tracking MSE

$$\lim_{n \to \infty} E\left\| h(n+k) - \hat{h}(n+k|n) \right\|_2^2$$

by tuning $\mu$ and the filters $P_k(q^{-1})$



# WLMS design


Select a model describing the dynamics of $h(n)$
Set the parameter-drift-to-noise ratio
$h(n) = \frac{C(q^{-1})}{D(q^{-1})} e(n)$
$\gamma$
Iterate
Design equations
Solve the spectral factorization
$r\beta(z^{-1})\beta(z) = \gamma C(z^{-1})C(z) + D(z^{-1})D(z)$
$\mu = 1 - \frac{1}{r} \quad 1 < r < \infty$
$r$ and $\beta$
$P_k(q^{-1}) = \frac{Q_k(q^{-1})}{Q_0(q^{-1})}\mathrm{I}$
Evaluate via
- simulations
- analytical MSE expressions
Solve Diophantine equations
$z^k \gamma C(z^{-1})C(z) = r \underbrace{Q_k(z^{-1})}_{\text{Unknown}} \beta(z) + zD(z^{-1}) \underbrace{L_k(z)}_{\text{Unknown}}$
$k = 1 \rightarrow Q_1(z^{-1}) = z(\beta(z^{-1}) - D(z^{-1}))$

# Selection and adjustment of fading models

In this case study, we consider AutoRegressive (**AR**) modelling, possibly with integration (**ARI**), of $h(n)$

Adjust AR models of arbitrary orders to the covariance function of $h(n)$

$$h(n) = \frac{\mathrm{I}}{D(q^{-1})}e(n) \qquad \mathrm{R_e} = E\{e(n)e^*(n)\}$$

Some design examples:

Second order AR (**AR2**)

$$D_2(q^{-1}) = 1 - 2\rho\cos\frac{\Omega_\mathrm{D}}{\sqrt{2}}q^{-1} + \rho^2 q^{-2}$$

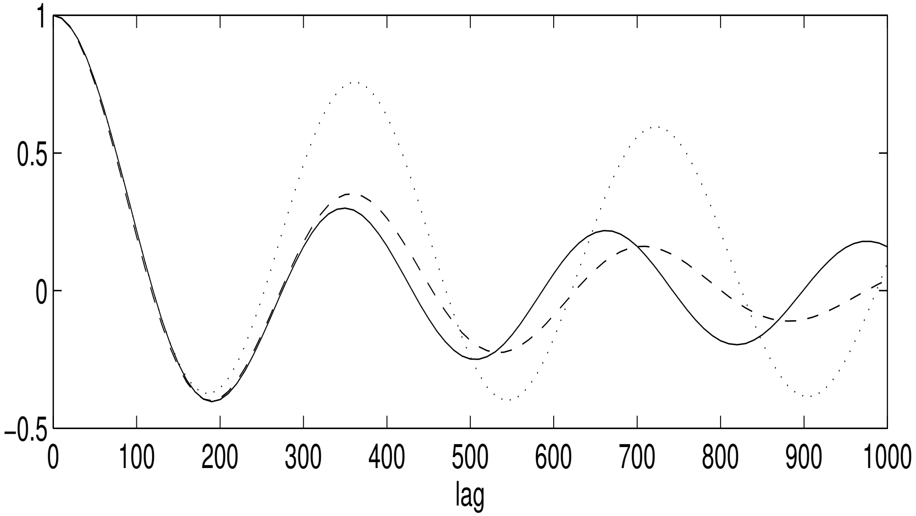
$$\rho = 0.999 - 0.1\Omega_\mathrm{D}$$

**AR2I**

$$D(q^{-1}) = (1 - q^{-1})D_2(q^{-1})$$

Adjusting **AR3** fading models $(\Omega_\mathrm{D} = 0.02)$

- True cov. function (solid)
- AR3 adjusted over 500 lags (dashed)
- AR3 adjusted over 150 lags (dotted)




Random walk (**RW**)

$$D(q^{-1}) = 1 - q^{-1}$$

$$P_\mathrm{k}(q^{-1}) = \mathrm{I}$$

?

# Low complexity fading models

First and second order fading models lead to simple design equations.

Prediction:

$$D(q^{-1}) = 1 + d_1 q^{-1} + d_2 q^{-2} \qquad C(q^{-1}) = 1$$

$$p = \frac{d_1 d_2 (1-\mu)}{1 + d_2(1-\mu)} \qquad 0 < \mu \leq 1$$

$$Q_{\mathrm{k}}(q^{-1}) = \mu \begin{bmatrix} 1 & q^{-1} \end{bmatrix} \begin{bmatrix} -d_1 & 1 \\ -d_2 & 0 \end{bmatrix}^{\mathrm{k}} \begin{bmatrix} 1 \\ p \end{bmatrix} \qquad \mathrm{k} \geq 0$$

$Q_0(z^{-1})$ is always minimum phase!

The step-size $\mu$ is obtained via $r$
or directly used as a design variable

Example: Integrated RW (**IRW**)

$$D(q^{-1}) = (1 - q^{-1})^2 = 1 - 2q^{-1} + q^{-2}$$

$$p = -\frac{2(1-\mu)}{2-\mu} \qquad \text{( “The pole” )}$$

$$Q_1(q^{-1}) = \mu((2+p) - q^{-1})$$

$$Q_0(q^{-1}) = \mu(1 + pq^{-1})$$

$$\boxed{P_1(q^{-1}) = \frac{(2+p) - q^{-1}}{1 + pq^{-1}}}$$

Smoothing: see conference proceeding

# Tools for Performance Analysis

The steady-state mean square one-step parameter prediction error

$$\mathrm{trP} \equiv E\left\| h(n) - \hat{h}(n|n-1) \right\|_2^2$$

ASSUMPTIONS:

- Signals and noise in the channel model are mutually independent and have zero means.
- The symbols and the noise are white sequences.
- The symbols have constant modulus (PSK-modulation).
- The number of channel coefficients M are < 3 and

$$(\mathrm{M}-1)\Sigma < 1$$

$\longrightarrow$

EXACT TRACKING MSE:

$$\mathrm{trP} = \frac{\Gamma + \mathrm{M}10^{-\mathrm{SNR}/10}\Sigma}{1-(\mathrm{M}-1)\Sigma}\mathrm{trR_h}$$

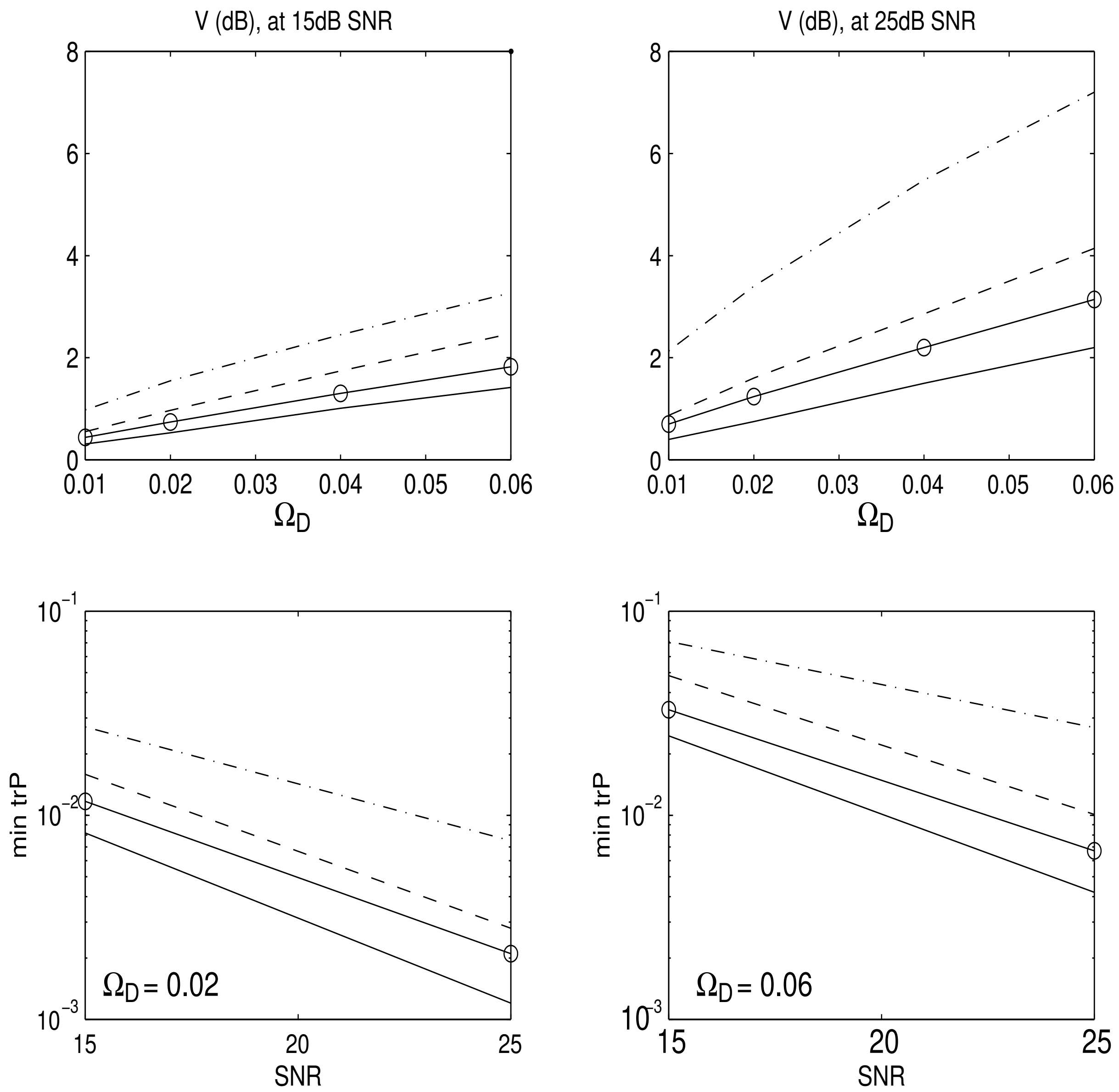
$$\Gamma = \frac{1}{2\pi}\int_{-\pi}^{\pi}\left|\frac{Q_1(e^{j\Omega})}{\beta(e^{j\Omega})}\right|^2 \mathrm{d}\Omega$$

$$\Sigma = \frac{1}{2\pi}\int_{-\pi}^{\pi}\left|\frac{D(e^{j\Omega})}{\beta(e^{j\Omega})}\right|^2 \frac{\phi_\mathrm{h}(\Omega)}{\mathrm{trR_h}}\mathrm{d}\Omega$$

Use the MSE expression for analysis and for the tuning of the WLMS

For a more general MSE expression, see conference proceeding

# MSE Performances




Performance indicator:

$$V = 10\log\left(\frac{\sigma_u^2 \mathrm{tr}P + \sigma_v^2}{\sigma_v^2}\right) \quad \text{(dB)}$$

WLMS based on fading model:

RW - dashed-dotted (LMS)
IRW - dashed
AR2 - circle
AR4 - solid

AR fading models adjusted to classical Rayleigh fading

$\Omega_D$ - Normalized Doppler frequency

# Simulation study

MSE optimized WLMS trackers are used together with Viterbi detectors for adaptive equalization of D-AMPS 1900 channels.

## Simulation conditions:

Forward link of IS-136

Two tap channel

$$y(n) = h_0(n)u(n) + h_1(n)u(n-1) + v(n)$$

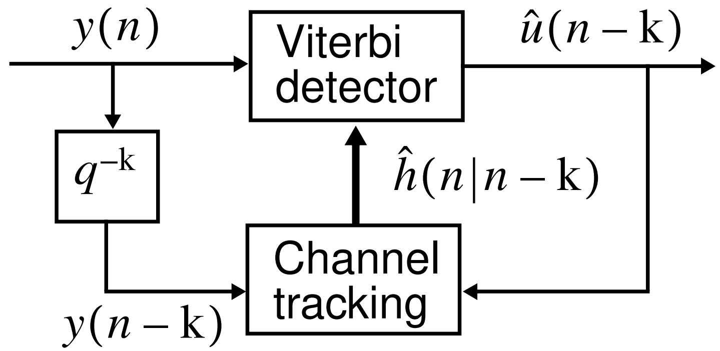
Classical Rayleigh fading, with equal tap powers

Indepedently fading taps (ideal synch)

Differential QPSK

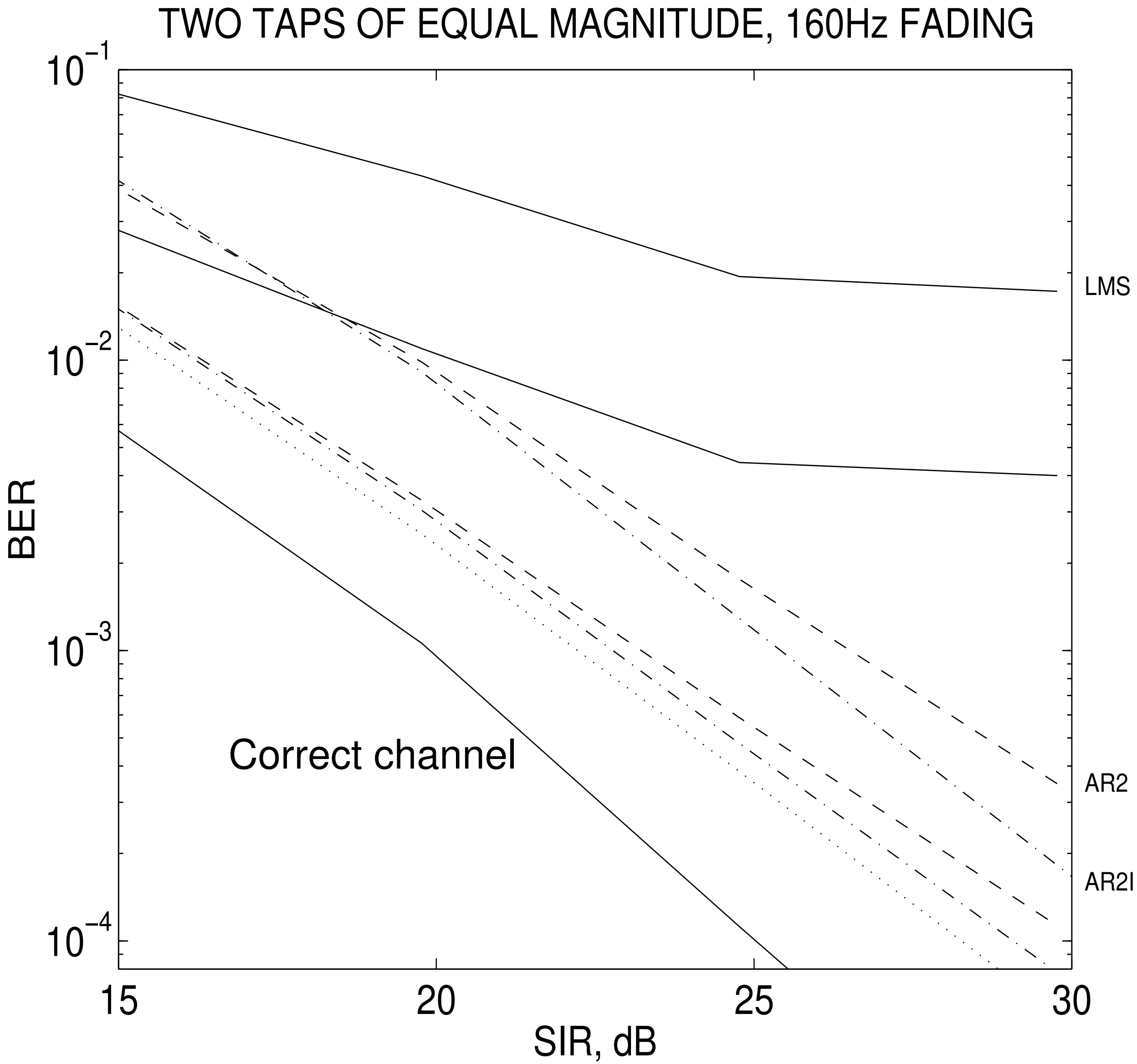
Burst synchronised interferers (colored)

## Adaptive equalization:




LMS best with decision delay k=3, otherwise k=4 is used.

# Uncoded Bit Error Rates




Detector performance:

The BER with correct channel is compared to WLMS based on different fading models.

WLMS based on fading model:

RW - solid (LMS)
AR2 - dashed
AR2I - dashed-dotted , dotted

Upper curves: decision-directed mode

Lower curves: WLMS feeded with true symbols.

Dotted curve: true symbols and correct initialization.

# Summary

- WLMS channel trackers based on AR2 and AR2I fading models provide good performance at low computational complexity in D-AMPS 1900.

- We obtain superiour BER, tracking MSE and prediction performance compared to LMS or RLS for two-tap channels. (For flat fading channels and single branch, not much can be gained by improved channel tracking.)

- Fading models need not to be very accurate: Channel estimators designed for one Doppler frequency and SNR (SIR) provide close to "tuned" performance also when used at lower speeds and higher SNR's. (See paper.)

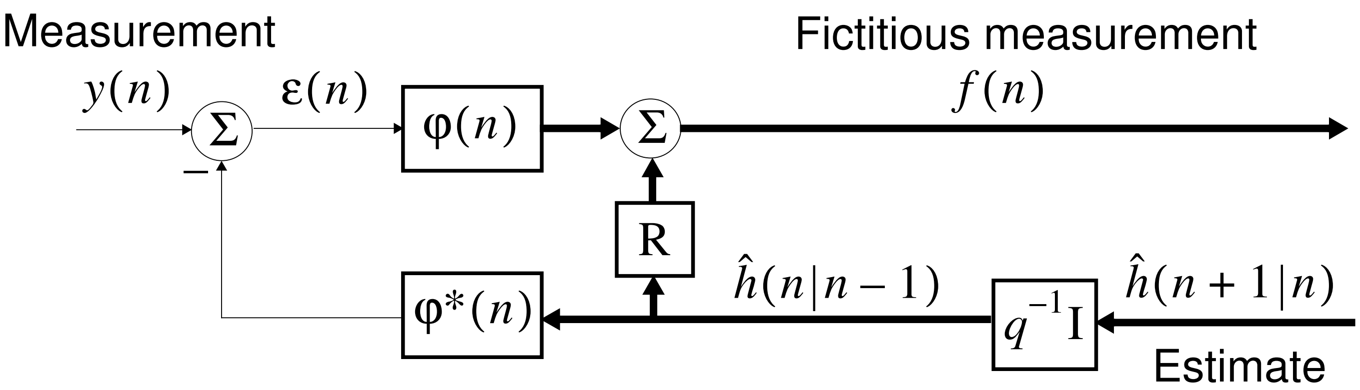
- WLMS is a special case of a class of adaptation algorithms with time-invariant gains, based on vector-ARIMA fading models

$$D(q^{-1})h(n) = C(q^{-1})e(n)$$

where $C$ and $D$ are not necessarily diagonal.
More general fading models and trackers provide higher performance e.g. in multi-user detectors with mobiles at differing speeds, see

www.signal.uu.se/Publications/abstracts/r001.html

# The fictitious signal model




$$
\begin{aligned}
f(n) &= \varphi(n)\varepsilon(n) + \mathrm{R}\hat{h}(n|n-1) \\
&= \varphi(n)y(n) - \underbrace{(\ \varphi(n)\varphi^*(n) - \mathrm{R}\ )}_{Z(n)}\hat{h}(n|n-1) \\
&= \varphi(n)(\ \varphi^*(n)h(n) + v(n)\ ) - Z(n)\hat{h}(n|n-1) \\
&= \underbrace{\mathrm{R}h(n)}_{\text{“Signal”}} + \underbrace{Z(n)(\ h(n) - \hat{h}(n|n-1)\ ) + \varphi(n)v(n)}_{\text{“Noise”}}
\end{aligned}
$$

$$
\eta(n) \equiv \underbrace{Z(n)(\ h(n) - \hat{h}(n|n-1)\ )}_{\text{“Feedback noise”}} + \varphi(n)v(n)
$$

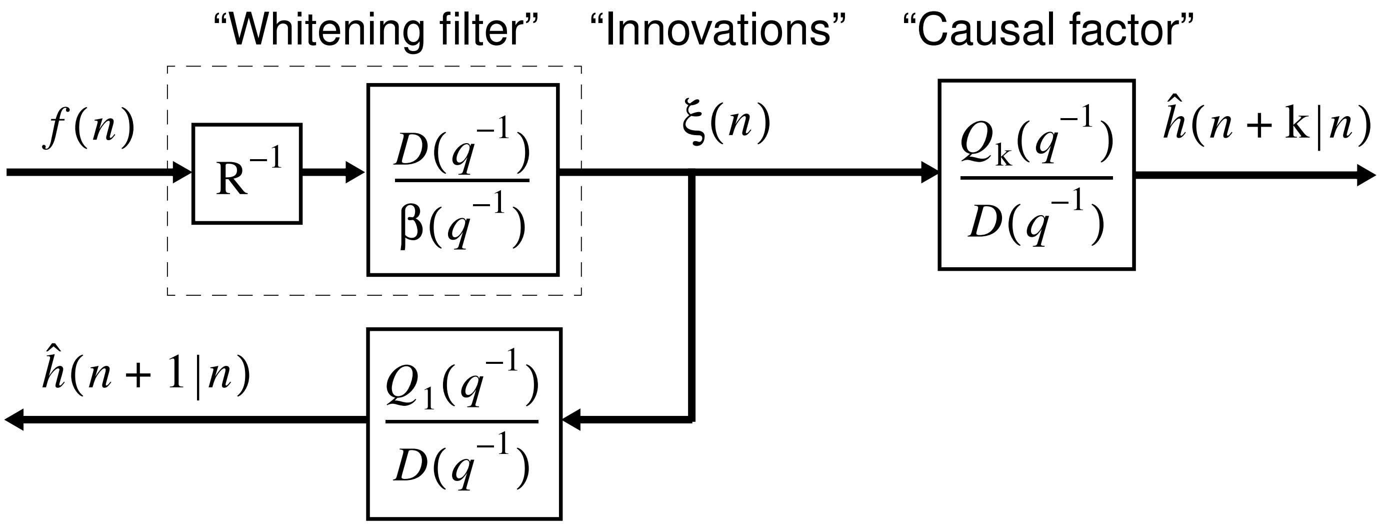
# Wiener filtering

**Assumption:**

The “noise” $\eta(n)$ is white and uncorrelated with the “signal” $\mathrm{R}h(n)$

The dynamics of the channel taps

$$h(n) = \frac{C(q^{-1})}{D(q^{-1})}e(n)$$




The parameter-drift-to-noise ratio:

$$\gamma = \frac{\mathrm{tr}\mathrm{R}_{\mathrm{e}}}{\mathrm{tr}\mathrm{R}^{-1}\mathrm{R}_{\eta}\mathrm{R}^{-1}} \qquad R_e = E\{\eta(n)\eta^*(n)\}$$