CHANNEL ESTIMATION AND DEMODULATION OF ASYNCHRONOUS CDMA SIGNALS IN FREQUENCY-SELECTION FADING CHANNELS

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ABSTRACT

In this paper, we propose an iterative scheme for joint tracking of fading channels and demodulation of multi-user CDMA signals operating asynchronously over multipath fading channels. We use the expectation maximization algorithm to track the time-varying channel and as a by-product we also achieve estimates of the input signals. We show that the maximum a posteriori estimates of the fading channels is iteratively computed by the Kalman smoother. The input signals are demodulated using a hidden Markov model smoother. Our scheme has computational complexity and memory requirement that grow linearly with the data length. Computer simulations illustrate the performance of our proposed detection and estimation method.

1 Introduction

Multiuser detection in CDMA communication channels was studied in great detail by Verdú in [1], where he devised both the minimum error probability detector and the maximum likelihood (ML) sequence detector for CDMA systems over Gaussian noise channels. Many suboptimal algorithms, that are computationally more practical than the optimal schemes, followed from the development of these optimal detectors, see *e.g.* [2, 3]. Recent work on multiuser detection in Rician and Rayleigh fading channels using ML and decorrelating detectors are found in [4, 5]. Joint tracking of fading channel and demodulation using per-survivor Kalman filtering can be found in [6, 7]. In [16], ML sequence estimation with unknown random phase or fading parameters treated as missing data is derived. In [9], the expectation maximization (EM) algorithm is used to compute the maximum a posteriori (MAP) sequence estimate of two-dimensional constellation signals transmitted over a Rayleigh fading channel. In [14], multiuser receivers iterate between the EM algorithm for amplitude estimation and multi-stage data detection. [12] uses the EM algorithm for amplitude estimation of direct sequence CDMA systems. The approach in [12] differs from the one in [14] in that the users' data are considered probabilistically as missing data.

In this paper, we perform iterative joint tracking of the fading channel and demodulation of a multi-user CDMA systems, where the signals are transmitted asynchronously over a multipath fading channel. The transmitted signals are modeled as finite-state Markov chains. We use the EM algorithm to yield estimates of the fading channels, that are optimal in a MAP sense, and as a by-product we also achieve estimates of the transmitted signals. We show that on each iteration of the EM, a closed-form solution of the tracking of the fading is achieved and that it is given by the Kalman smoother. The demodulation is performed by using a hidden Markov model (HMM) smoother. Our approach is different from previous work, since we compute soft decisions on the data symbols using a HMM estimator, while at the same time we compute MAP estimates of the time varying fading channel.

2 Problem Description

In this paper, we assume that the multiuser CDMA system operates over a multipath channel subject to fading. The propagation delays are assumed to be known a priori. Furthermore, it is here assumed that the number of echo paths S, the number of users K, and the number of symbols N, are known and fixed. The input signals are assumed to belong to the finite symbol alphabet $\{0, 1, \dots M-1\}$, where M is known. We use a finite-state Markov chain with known transition probabilities to model the sequence of input signals. The input data are modulated using PSK. The kth user's modulated signal is given by

$$s_k(t) = \sqrt{2P_k} e^{j\phi_k} \sum_{n=1}^{N} e^{j\frac{2\pi}{M} m_{n,k}} \times \sum_{l=0}^{L-1} \mathrm{rect}_{T_c}(t - lT_c - nT) d_k(l), \; -\infty < t < \infty, \tag{1}$$

where $m_{n,k}$ is the nth symbol from the kth user, T is the symbol duration, $T_c = T/L$ is the chip duration, and

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L is the number of chips per symbol. Here, $d_k(l), l = 0, \ldots L-1$, denotes the known code sequence of the kth user. P_k and ϕ_k are the transmitted power and the carrier phase relative to the local oscillator, respectively. Furthermore, $\mathrm{rect}_{T_c}(\cdot)$ is a rectangular pulse of duration T_c, given by

$$\mathrm{rect}_{T_c}(t) = \begin{cases} 1, & \text{if } 0 \le t \le T_c \\ 0, & \text{otherwise} \end{cases} \quad (2)$$

The symbol $m_{n,k} \in \{0, 1, \ldots M-1\}$ is modeled as a finite-state Markov chain. The transition probability matrix of $m_{n,k}$ is denoted by $\Pi^{(k)} = \left[p_{ij}^{(k)}\right]$, where for $i, j \in \{0, 1, \ldots, M-1\}$, $k = 1, \ldots K$, $p_{ij}^{(k)} \triangleq \Pr(m_{n+1,k} = j | m_{n,k} = i)$, is the probability that the kth user's transmitted signal at time instant $n+1$ is j given that it was i at time instant n. The following restrictions on $p_{ij}^{(k)}$ must hold: $0 \le p_{ij}^{(k)} \le 1, \quad \forall i, j \in \{0, 1, \ldots, M-1\}$, $k = 1, \ldots K$, and for each $i \in \{0, 1, \ldots, M-1\}$, $k = 1, \ldots K$, $\sum_{j \in \{0, \ldots, M-1\}} p_{ij}^{(k)} = 1$. Furthermore, we assume that the messages between the users are statistically independent.

The modulated signals are subject to propagation delays, time-varying attenuations and phase shifts during the transmission over the multipath fading channel. The received signal is given by

$$r(t) = \sum_{k=1}^{K} r_k(t) + w(t), \ -\infty < t < \infty, \quad (3)$$

where $r_k(t)$ is the received, attenuated and delayed signal of the kth user and $w(t)$ is assumed to be white complex Gaussian noise with zero mean and a two-sided power spectral density σ^2. The signal $r_k(t)$ is given by

$$r_k(t) = \sum_{p=1}^{S} \alpha_{k,p}(t) \ s_k(t - \tau_{k,p}), \quad (4)$$

where $\alpha_{k,p}(t)$ denotes the attenuation and the phase shift and $\tau_{k,p}$ is the propagation delay of the kth user via the pth propagation path.

The received continuous-time signal is converted into discrete-time by sampling the output matched to the chip waveform, which is a rectangular pulse for PSK [11]. As in [11], we assume that the time-delays are less than the symbol period, *i.e.* $\tau_{k,p} \le T$. Using this assumption, each observation will contain the end of the previous symbol and the beginning of the current symbol for each user and path. Furthermore, we assume that $\alpha_{k,p}(t)$ is constant during one symbol period, *i.e.* $\alpha_{k,p}(t) = \alpha_{n,k,p}$ for $t \in [nT, (n+1)T)$. Extending the results in [11], it can be shown that the discrete-time vector, $y_n \in \mathcal{C}^L$, which is the nth vector output from the integrate-and-dump chip is given by

$$y_n = \sum_{k=1}^{K} \sum_{p=1}^{S} \xi_{n,k,p} \ c_{n,k,p}(m_{n-1,k}, m_{n,k}) + w_n \quad (5)$$

where

$$c_{n,k,p}(m_{n-1,k}, m_{n,k}) = e^{j\frac{2\pi}{M} m_{n-1,k}} \rho_{k,p} + e^{j\frac{2\pi}{M} m_{n,k}} \varrho_{k,p} \quad (6)$$

with

$$\rho_{k,p} = [(1-\eta_{k,p}) a_k^r(\nu_{k,p}) + \eta_{k,p} a_k^r(\nu_{k,p}+1)] \quad (7)$$
$$\varrho_{k,p} = [(1-\eta_{k,p}) a_k^l(\nu_{k,p}) + \eta_{k,p} a_k^l(\nu_{k,p}+1)] \quad (8)$$

and $w_n \in \mathcal{C}^L$ where each element is a zero mean white complex Gaussian noise sequence with variance σ^2/T_c and

$$\xi_{n,k,p} = \alpha_{n,k,p} \sqrt{2P_k} e^{j\phi_k}. \quad (9)$$

The time delay $\tau_{k,p}$ is separated into an integer part $\nu_{k,p} \in \{0, 1, \ldots, L-1\}$ and a fractional part $\eta_{k,p} \in [0, 1)$,

$$\frac{\tau_{k,p}}{T_c} \mathrm{mod}(L) = \nu_{k,p} \quad (10)$$

and

$$\eta_{k,p} = \tau_{k,p} - \nu_{k,p} T_c. \quad (11)$$

Finally,

$$a_k^r(\nu) \triangleq [d_k(L-\nu) \cdots d_k(L-1) \ 0 \cdots 0]', \quad (12)$$

$$a_k^l(\nu) \triangleq [0 \cdots 0 \ d_k(0) \cdots d_k(L-\nu-1)]'. \quad (13)$$

State-Space Model:
In order to simplify the derivation of the explicit expression for the MAP solution of the tracking of the fading characteristics, we choose to rewrite the above described CDMA system model Eq. (5) in a state-space form. If x_n and C_n form the following matrices

$$x_n = [\xi_{n,1,1}, \ldots, \xi_{n,1,S}, \xi_{n,2,1}, \ldots, \xi_{n,K,S}]',$$
$$C_n = [c_{n,1,1}(m_{n-1,1}, m_{n,1}), \ldots, c_{n,K,S}(m_{n-1,K}, m_{n,K})],$$

then the observation vector Eq. (5) can be written in the following matrix notation

$$y_n = C_n H x_n + w_n, \quad (14)$$

where H is a known matrix (can be computed from the second order statistics of the channel [7]). We choose to model the time variation of the attenuation and phase shift as a linear Gauss Markov process of the form

$$x_n = A x_{n-1} + B v_n \quad (15)$$

where A and B are known matrices (they can be computed from the second order statistics of the channel [7]), and v_n is a vector of iid complex Gaussian processes with a known covariance matrix R. Finally, let $x_0 \sim N(x_{0|0}, P_{0|0})$, where $x_{0|0}$ and $P_{0|0}$ are the initial channel mean and initial state covariance matrix, respectively.

Eqs. (14,15) form a state space model of the fading CDMA system. Note that the observation matrix, $C_n H$, is time-varying and is a function of the current and previous transmitted user signals $m_{n,k}$ and $m_{n-1,k}$, respectively.

Notation: Let the sequence of measurements $(y_1, \ldots, y_N)$ and channel states $(x_1, \ldots, x_N)$ be denoted as Y and X, respectively. Furthermore, let Y_q^p denote $(y_q, \ldots, y_p)$ and Y_n denote $(y_1, \ldots, y_n)$. Let M_k denote the symbol sequence $(m_{1,k}, \ldots, m_{N,k})$ of the kth user. Finally, let $m_n = \{m_{n,k}; k = 1, \ldots, K\}$ and $M = \{M_k; k = 1, \ldots, K\}$.

2.1 Objectives

In this paper, we aim at performing joint tracking of the fading channel, *i.e.*, estimating X, and demodulating the transmitted data sequence of the K users, *i.e.*, estimating M. The tracking of the fading characteristics should be optimal in a MAP sense. Thus, our objectives are as following. Given the observed data Y:

1. **Channel State Estimation:** Compute the MAP estimate of the fading channel, defined in (9), for each user k and each path p as follows

$$X^{MAP} \triangleq \arg\max_X f(X|Y), \tag{16}$$

where $f(\cdot)$ is the probability density function.

2. **Input Sequence Estimation:** Estimate the transmitted input sequence $m_{n,k}$, for $n = 1, \ldots, N$ and $k = 1, \ldots, K$, as follows

$$\hat{m}_{n,k} = \arg\max_{m_{n,k}} f\left(m_{n,k} \mid Y, X^{MAP}\right) \tag{17}$$

In the following section, we give details for our proposed iterative scheme for joint tracking of the fading characteristics and demodulation of the transmitted signals.

3 Channel State and Input Sequence Estimation Scheme

The EM algorithm [13] is normally used as an iterative parameter estimation scheme for extracting the mode of the likelihood function or computing the mode of the posterior distribution for incomplete data models. Here, we use the EM algorithm to yield the MAP channel state estimates, X^{MAP}. As a by-product of the EM algorithm, we achieve estimates of the input signals.

The EM algorithm obtains X^{MAP} by generating a sequence of state estimates $\{X^{(l)}\}$, where the superscript (l) denotes the lth iterate, from an initial state sequence estimate $X^{(0)}$. The appealing property of the EM algorithm is that the posterior density increases monotonically, *i.e.* $f(X^{(l+1)}|Y) \geq f(X^{(l)}|Y)$ with equality holding at the stationary points (local minima, maxima and saddle points) of the posterior distribution, given mild regularity conditions [10]. See [17] for details. The convergence of X to a stationary point, whether it is local maximum, local minimum or a saddle point, depends on the choice of the starting point $X^{(0)}$. Thus, it is usually recommended that several EM iterations are tried with different starting points [10].

We now present the details of the EM algorithm for computing MAP estimates of the channel states.

Expectation Step: Evaluate

$$\mathcal{Q}\left(X, X^{(l)}\right) \triangleq \mathrm{E}\left\{\ln f(Y, X, M) | Y, X^{(l)}\right\}. \tag{18}$$

The expectation step requires the evaluation of the following probabilities

$$\gamma_n^{(l)}(i_m, j_m) \triangleq \Pr\left(m_{n-1} = i_m, m_n = j_m | Y, X^{(l)}\right), \tag{19}$$

where $i_m, j_m \in \{0, 1, \ldots, M-1\}^K$. The probabilities in (19) are efficiently computed via the forward-backward recursions of an HMM smoother according to [15].

Maximization Steps: Ignoring terms in Eq. (18) irrelevant to the maximization with respect to X, expanding the terms and re-arranging, we compute the updated channel state estimate $X^{(l+1)}$ as follows

$$\begin{aligned} X^{(l+1)} &= \arg_X \max Q(X, X^{(l)}) \\ &= \arg_X \min \sum_{n=1}^{N} \Big\{ (x_n - Ax_{n-1})^H (BRB^H)^{-1} (x_n - Ax_{n-1}) \\ &\quad + \left(\overline{z_n}^{(l)} - Hx_n\right)^H \left(\overline{R}^{(l)}\right)^{-1} \left(\overline{z_n}^{(l)} - Hx_n\right) \Big\} \end{aligned} \tag{20}$$

where $\overline{z_n}^{(l)}$ is the synthetic measurement and $\overline{R_n}^{(l)}$ is the the synthetic measurement error covariance matrix, given by

$$\overline{z_n}^{(l)} = \left(\overline{C_n^H C_n}^{(l)}\right)^{-1} \left(\overline{C_n}^{(l)}\right)^H y_n \tag{21}$$

$$\overline{R}^{(l)} = \frac{\sigma^2}{T_c} \left(\overline{C_n^H C_n}^{(l)}\right)^{-1} \tag{22}$$

where the synthetic observation matrix $\overline{C_n}^{(l)}$ and $\overline{C_n^H C_n}^{(l)}$ are respectively given by

$$\overline{C_n}^{(l)} = \sum_{i_m, j_m} C_n \gamma_n^{(l)}(i_m, j_m) \tag{23}$$

$$\overline{C_n^H C_n}^{(l)} = \sum_{i_m, j_m} C_n^H C_n \gamma_n^{(l)}(i_m, j_m) \tag{24}$$

The minimization in Eq. (20) is efficiently computed using a fixed-interval Kalman Smoother operating on the following state-space model

$$x_n = Ax_{n-1} + Bv_n \tag{25}$$

$$\overline{z_n}^{(l)} = Hx_n + \overline{w_n}^{(l)} \tag{26}$$

where $\overline{w_n}^{(l)} \sim N\left(0, \overline{R}^{(l)}\right)$. The updated channel states on the $(l+1)$th iteration are given by the following

Forward Pass: for $n = 1, 2, \ldots, N$

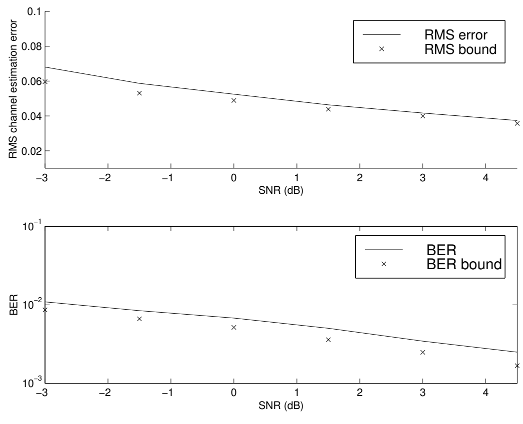
$$\begin{aligned}
x_{n|n-1} &= Ax_{n-1|n-1} & (27)\\
P_{n|n-1} &= AP_{n-1|n-1}A^H + BRB^H & (28)\\
G_n &= P_{n|n-1}\left(P_{n|n-1} + \overline{R_n}^{(l)}\right)^{-1} & (29)\\
x_{n|n} &= x_{n|n-1} + G_n\left(\overline{z_n}^{(l)} - Hx_{n|n-1}\right) & (30)\\
P_{n|n} &= P_{n|n-1} - G_n\left(P_{n|n-1} + \overline{R_n}^{(l)}\right)G_n^H & (31)
\end{aligned}$$

Backward Pass: for $n = N-1, \ldots, 1$

$$\begin{aligned}
J_n &= P_{n|N}A^H P_{n+1|n}^{-1} & (32)\\
x_n^{(l+1)} &= x_{n|n} + J_n(x_{n+1}^{(l+1)}) - x_{n+1|n} & (33)\\
P_{n|N} &= P_{n|n} + J_n(P_{n+1|N} - P_{n+1|n})J_n^H & (34)
\end{aligned}$$

3.1 Numerical Results

1000 Monte Carlo simulations have been carried out to evaluate the performance of our proposed demodulation and estimation scheme. The number of users was chosen to be $K = 2$ and the number of multi-paths for each user was $S = 2$. The constellation size was $\mathcal{M} = 2$. As in [9], we used an AR(1) model to describe the fading channel. We selected $A = 0.99I_{4\times4}$. The number of chips per symbol was $L = 31$. The spreading sequences $d_k(l)$ and the channel parameters were randomly chosen from one simulation to the next. The algorithm was iterated until convergence or up to a maximum number of 5 iterations. In Fig. 1, the RMS and BER are depicted, respectively. The RMS bounds are computed using a Kalman smoother and assuming known transmitted data. The BER bounds are computed assuming known channel fading.



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